THE

STUDENT'S HANDBOOK

SYNOPTICAL AND EXPLANATORY

OF

MR J. S. MILL'S SYSTEM OF LOGIC

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PREFACE.

THE design of this Handbook is to facilitate as much as possible the study of Inductive Logic,—particularly as represented in Mr J. S. Mill's volumes on the subject. It is therefore, in the main, an epitome of that work, the arguments being condensed and summarised, the necessary explanations being given wherever it seemed likely that a student would feel any difficulty, and the whole being so arranged that the connexion and relative importance of the different topics discussed may be recognised at a glance. The single aim of the author has been to render the work what its name imports—a Handbook to aid the careful study of the original by furnishing the

Those whose logical reading has been confined to Whately or the common manuals of the science, may perhaps be not a little perplexed, on directing their attention for the first time to the study of Mill, by the total difference in the manner in which the entire subject appears to be treated. Many topics which are entirely omitted, or very slightly treated, in the most popular logics, or if mentioned, are mentioned only to be expressly excluded from the domain of the science, are elaborately discussed by Mr Mill; who, on the other hand, passes over, with scarcely any notice, many subjects which occupy a large space in the treatises of most other logical writers. Some of these differences are merely such as would occur between any two independent thinkers discussing the same subjects; some are connected with differences of opinion on certain metaphysical points, which, though themselves no part of logical science, necessarily modify the views which are taken of logical questions; but in general they depend upon a more fundamental cause, a due consideration of which will not only often explain the apparent an-

and hopelessly entangled questions with which Logical Science is concerned. The explanation referred to will be found to a great extent to be involved in the distinction between what may be termed respectively "*Objective*" and "*Subjective Inference*,"—a distinction of great importance, and one which it is essential that the student should thoroughly comprehend.

In "*Objective Inference*" the fact stated in the conclusion is a bonâ fide *new truth*, a distinct fact, and not merely part of the same fact or facts stated in the premisses. Thus, if we find that half-a-dozen pieces of loadstone possess each the property of attracting iron, and hence infer that a seventh piece which we have not tried will also manifest the same property, it is perfectly clear that this last fact is something new, and by no means included in the previous facts (that the six loadstones attract iron) which form the premisses of our conclusion. In such a case, as in all cases of Objective Inference, the conclusion follows in virtue of *a law of External Nature* (hence the designation "Objective"), and not by a mere law

and whether it does or does not will evidently be a mere question of physical law. A consequence of this is, that such inferences cannot be expected in symbols in such a way that the conclusiveness of the argument is evident from the *mere form*,—i.e., whatever meaning we choose to assign to the symbols. "Objective Inference" is the "Induction" of Mill; with other logical writers it is usually spoken of as "Material Induction," and is not only contrasted with what they call "true logical Induction" (which we shall find to be the same with Mr Mill's "Mere Verbal Transformation"), but is by them expressly excluded as a subject whose consideration ought to form no part of Logical Science.

"*Subjective Inference*," on the other hand, affords a contrast in all these respects. It is, in short, *an explicit statement* of a fact drawn from premisses in which it was in reality *implied*, so that the mind, being in possession of the premisses, can, by a mere expression in words, evolve the conclusion being

is clear that this last is really involved in the previous statement, and we could not believe the former and disbelieve the other without violating a law of the mind itself—without, in fact, being guilty of a contradiction. Hence this form of Inference may be expressed in *symbols*, in such a way that the inference may be seen to follow from the *mere form* of the expression. Thus, putting A for "men," B for "mortal" (beings), C for Cæsar, we have—

All A is B
C is A
therefore C is B.

Whatever A, B, C may stand for, if we assent to the premisses in such a case as this, we cannot refuse our belief to the conclusion without a contradiction. The terms "*formal*" and "*subjective*" inference are, in fact, convertible. The "*Syllogism*" and the so-called "*Immediate Inferences*" are the principal forms which subjective inference assumes.

If now the distinction which has been pointed out be understood, its application

inference—in other words, to the Syllogism and certain subordinate processes; objective inference they refuse to recognise as coming within the domain of the science at all, and relegate it to the limbo of the "extra-logical." Whately over and over again asserts that all reasoning is Syllogism, and that to the consideration of this the duty of the logician is strictly confined; that objective inference—the process by which we arrive at *new truths*—is altogether foreign to the subject, and that it is impossible to lay down rules for it.

Mr Mill, on the other hand, takes a view as directly opposed to this as possible; for he refuses to recognise what we have above distinguished as subjective inference—as being a real inference at all in the proper sense of that word--but views the conclusion in a Syllogism as being rather an *interpretation* of, than a deduction from, the so-called premisses.

Further, he regards objective inference, or true induction, as not only a part, but by far the most important part of the province of Logic, and the greater portion of his work is occupied with the consideration of *this* process,—its laws, its methods, and the general conditions of its validity—almost to the exclusion of those different forms of formal inference

which most other logicians put forward as the only possible formulæ of reasoning.

Now it is scarcely to be wondered at that authors who differ so fundamentally in their views of this subject, should differ widely in their mode of treating it. The truth seems to be this—Logic is undoubtedly the Science of Inference, and *not merely of one, but of every form* in which that mental operation can be presented to us. Such is the principle upon which Mill proceeds—that Induction is properly within its province as well as Syllogism. Whether we decide to apply the term inference to the latter or not,—or whether we are content to follow Mr Mill in regarding the Syllogism as simply a formal mode of interpreting and applying general propositions—in any case, its consideration forms an important division of the general Science of Logic. It will thus be evident that the great apparent discrepancies between authors like Whately and Hamilton on the one hand, and Mill on the other, are in reality to a great extent not differences at all; they are treating of different branches of the total Science of Inference, and apart from inconsistencies necessarily connected with the different metaphysical tenets of the various schools, the general body of

their logical doctrines may be combined into one system, each having its distinct function in the generation of scientific belief. A *complete* treatise on Logic would necessarily incorporate material from them all,—their systems are not, *as wholes*, inconsistent, but complementary.

It only remains to add a word of explanation in reference to the arrangement of the present work. Each chapter will be found subdivided into sections, marked by Roman numerals in the margin; in general each such section is limited exclusively to the consideration of a single topic. In Book V., on the Fallacies, Whately's divisions have been introduced, each under its proper heading, in the exhaustive classification of Mill; and those passages throughout the work which have not the authority of a direct statement in Mr Mill's volumes, are marked by inclosure in dark, square brackets.

CONTENTS.

INTRODUCTION.

DEFINITION AND SCOPE OF LOGIC.

MILL, after remarking that the Definition of any progressive Science must necessarily be provisional, gives two Definitions of Logic, which he criticises, before ennunciating his own.

1*st Definition.*—"Logic is the Science and Art of Reasoning."

[Whately added the term "*Science*" to this definition, and properly so. By "*Science*" is here meant the analysis of the mental processes which take place whenever we reason; and by "*Art*," the rules for properly conducting the process founded upon that analysis.]

[*Meaning of term "Reasoning."* In its restricted sense, it is equivalent to Syllogizing or Ratiocination; in a more extended and proper sense, it is simply to infer any assertion from assertions previously admitted.]

Criticism of this Definition.—This definition of
of argumentation

of Names and Definitions), and *Accuracy of Classification* are almost always reckoned amongst the objects of that science; for we find—

(*a.*) *The view of professed Logicians is more extensive.* Most authors, dividing Logic into three parts, treat in the first two of Names or Notions and Propositions (under one or other of which heads they include Definition, Division, &c.), and in the third part only do they discuss Reasoning.

(*b.*) *The popular view of Logic, also, is more extensive.* In common discourse we hear as often of "a logical arrangement," or of expressions "logically defined," as of conclusions logically deduced from premisses.

2d Definition.—"Logic is the Science which treats of the operations of the Understanding in the pursuit of Truth."

Criticism.—This definition is *too wide*, because—

(*a.*) Logic has nothing to do with one class of Truths,—those of *Intuition*.

[Truths are known to us in two ways:—

1. By our immediate consciousness, *i.e.*, by Intuition.
2. By Inference.

The truths known to us by Intuition are such as are involved in our Feelings, bodily or mental,—that I have such and such a feeling; that I am experiencing such and such an emotion or mental state; that I am conscious, for instance, that I feel hot, or cold,—is known to me beyond the possibility of doubt. No Science is required for the purpose of testing the validity of our belief in such truths as these; no Art can possibly render our knowledge of them more certain.

Rapid and unconscious inferences must not, however, be confounded with true intuitions.—We may really *infer* what we fancy that we actually see or feel; thus a certain combination of sensations of form and colour, imprinted as a picture on the retina, have always been infallible marks of the presence of an external object, as my father; and hence, whenever I experience such a combination of sensations, I *infer* his presence, though I may do so wrongly, as in delirium or spectral illusion.]

(*b.*) It would introduce into Logic many questions with which it has no direct concern, many metaphysical inquiries especially.

[Such inquiries as:—

What is the nature of Perception, Memory, Belief, or Judgment? Are God and Duty realities, the existence of which is manifest *à priori?* and so forth,—are all connected with "the pursuit of truth," but are foreign to Logic.]

Province of Logic and its Relation to Knowledge in general.—The province of Logic, then, must be restricted to *that portion* of our knowledge which consists of *Inferences* from data. Every proposition, therefore, which is believed as being an inference from something else, comes within the scope of the principles and tests furnished by Logic.

The function of Logic ... part of our

matter, furnishes *the evidence itself*. Logic supplies the principles or rules for *the estimation of the (worth of it as) evidence*.

[*The Logic of Science the same as the Logic of common life. To draw inferences has been said to be the great business of human life.*—By far the greater portion of our knowledge is indubitably matter of inference; so that not only Science, but the great bulk of human beliefs generally, are amenable to logical tests. The business of the physician, general, magistrate, &c., is chiefly to draw inferences from data, and to act according to the conclusion therefrom.]

[*Utility of Logic.*—"If a Science of Logic exist, that Science must be useful; if there be rules to which every mind conforms when it infers rightly, it seems self-evident that a person is more likely to act in accordance with those rules, if he know them, than if he be ignorant of them."]

Mill's Definition.—"Logic is the Science of the operations of the Understanding, which are concerned in, or are subservient to, the Estimation of Evidence."

[In other places Mill gives the following verbally-different forms of this Definition:—
(1.) "Logic is the Science of the investigation of Truth by means of Evidence."
(2.) "Logic is the Science of Inference."]

Its main subject, then, is Inference; amongst the subsidiary we may notice:—

1. Theory and *Uses of Names and Propositions*—

form; and (2.) to marshal our facts in a clear order.

[*Analysis of Instruments necessary.*—The Analysis of the Instruments (Names and Propositions) we employ in the investigation of Truth is part of the Analysis of the investigation itself; since no art is complete unless another art, that of constructing and adjusting the necessary tools, is embodied in it.]

[*How far such Analysis must be carried.*—The Analysis of the process of Inference, and of the processes thereto subordinate, need only, for the purposes of Logic, be carried far enough to enable us to discriminate between a correct and incorrect performance of those processes.]

BOOK I.

NAMES AND PROPOSITIONS.

CHAPTER I.

INTRODUCTORY.

[THE remarks on language in this chapter are, *par excellence*, applicable to *general names* (whether consisting of one or more words), the parts of language with which, and the copula, Logic is chiefly concerned.]

I. *The necessity, in Logic, of commencing with an Analysis of Language.*

"Logic is concerned with a portion of the art of thinking. Language is the principal help and instrument of thought, and any imperfection in this instrument is confessedly especially liable to confuse and impede the main process, and to destroy all confidence in the result."

Answer as far as Reasoning is concerned.—Since inference is a process usually carried on by means

of words, and in complicated cases can be carried on in no other way, a thorough insight into the import of language is evidently an essential preliminary to a correct performance of that process.

II. *Theory of Names especially, why a necessary part of Logic.*

Since Logic is the science of proof, or that by which we test the validity of the inference of a proposition from propositions already admitted,—it is evident that a clear view of the *Import of Propositions* is essential. To this an analysis of Names—the chief elements of a proposition—is an indispensable preliminary.

III. *Why Names must be studied before Things.*

If one should commence with the study (*i.e.*, examination and classification) of Things, without using the aid of established Names,

(1.) No varieties of things would of course be included but those personally recognised by the individual observer; and this is to discard the results of the labour of all preceding observers; and

(2.) Even after his personal examination of things, it will remain necessary to examine

names, to be sure that nothing is omitted which ought to be included.

(3.) But by beginning with the classifications recognised in common language, and thus using names as a clue to the things, we bring before us at once all the differences and resemblances amongst objects which have been recognised, not by a single inquirer merely, but by the collective intelligence of mankind.

IV. *Definition and Initial Analysis of Propositions.*

A Proposition is a sentence in which something is affirmed or denied of something.

It is formed by putting together two names; and consists of the two names—the subject and predicate, connected by the copula.

The *Subject* is the name denoting that of which something is affirmed or denied.

The *Predicate* is the name denoting what is affirmed or denied of subject.

The *Copula* is the sign denoting that there is an affirmation or denial.

CHAPTER II.

Names.

"A Name is a word or set of words, serving the double purpose of—(1.) A mark to recall to ourselves the likeness of a former thought; and (2.) Of a sign to make it known unto others."

I. *Are Names the names of Things or of our Ideas of Things?*

Names are properly the names of things.

Names are names of our ideas of things in this sense only,—that the idea alone, and not the thing itself, is recalled by the name, or imparted to the hearer.

But names are not only intended to make the hearer *conceive what we conceive*, but also to *inform him what we believe;* and therefore it seems proper to consider a name as *the name of that which we intend to be understood by it* on the particular occasion when we make use of it,—that, in short, concerning which we intend to give information,

3. Of attributes which have attributes, thus, "fault" is the name of some attribute which has the quality—"*causing inconvenience*."]

6. A *Non-connotative Name* denotes a subject only, or an attribute only.

7. A *Connotative Name* denotes a subject (or subjects), and implies or involves an attribute (or attributes).

[Thus the name "man" means or connotes certain attributes—animality, rationality, upright form, &c. Other logicians would say the *idea* of "man" includes or "comprehends" the idea of animality, &c.

The best mode of determining whether a name connotes a given attribute is to ask, Whether, if that attribute were removed, the name would still be applied to the subjects? Does "man" connote mortality? The test is, should we apply the name "man" to beings exactly like men in other respects, but not mortal?]

Connotative Names are :—

(1.) *All* concrete general names.

(2.) *Abstract* names, groups 2 and 3.

(3.) *Certain singular* or individual names,—that is, *names not per se* singular, but determined somehow *to a single* individual; in fact, names which *describe* an individual. Thus the "only son of Jones," the "present premier," &c.

Non-connotative Names are :—

(1.) *Proper* names.

(2.) *Names of* attributes which have no attributes.]

(*See* Appendix.)

8. Posi...

9. *Negative Name*—the negation or contradictory of a positive name, "*not man*," "*not tree*" (includes everything else except what is denoted by positive name).

10. *Privative Name*—equivalent to a positive and negative name together, being the name of something which might be expected to have a certain attribute, but has it not. Thus "blind" = might be expected to see, but does not.

Connotation of these Names.—The *negative* connotes the *absence* of the attributes connoted by corresponding positive; the *privative* connote (1.) the absence of certain attributes, (2.) the presence of others from which the presence also of the former might be expected.

11. *Relative Names.*—A name is said to be relative when over and above the object it denotes, it implies in its signification the existence of *another object*, also deriving its denomination from the same fact or series of facts which is the ground of the first name.

Thus take any pair of relative names, as *parent*, *offspring*. Now it is clear that we cannot speak of a *parent* without implying existence of *offspring*, and *vice versâ*. When we call A the parent of B, we understand that a certain series of events or phenomena have happened, in which A and B are both involved. This *series of facts* is implied whenever we speak ...

stands to those facts is different from the attitude of *offspring* to them.

The *characteristic property*, then, of relative names, is that they are always given *in pairs;* every relative name (as *father, son; like, unlike; equal, longer*), which is predicated of an object, supposes another object or objects, of which we may predicate either the same name (as "*consort*"), or another relative name, said to be the "correlative" of the former (as *offspring* and *parent*).

The *Connotation of a relative name*, then, consists of (1.) the fact, or series of facts, which constitutes the relation; and (2.) the attitude or position in which the object denoted by the name stands to those facts. Two correlative names have the first part of the connotation in common; they may, or may not, have the latter (*i.e.*, they may or may not occupy an identical attitude with reference to the facts implied in the relation, but, in either case, they must both connote those facts).

[Thus *parent—offspring* both connote the facts which connect them, but they each connote a different attitude or position in regard to those facts; while *consort* implies another *consort*, both objects holding the same position to the facts which connect them.]

12. *Non-relative Names* include, of course, all but relative names.

13. Uni...

A name is used equivocally when it is used in distinct senses.

A name is used analogically when it is used in a signification somewhat similar to its primary and proper meaning.

CHAPTER III.

The Categories.

Mr Mill first examines and criticises the arrangement of the Categories given by Aristotle and the majority of Logicians; and afterwards proposes an arrangement of his own.

The word "Thing" is used in its widest sense throughout this chapter. It is not limited to material or really existing objects, but includes every object of Sense or Imagination of which we can become conscious, or to which a name can be given; thus *God, a spirit, a centaur, an attribute* (as *blueness, hardness*), or *a feeling* (as of *pain, anger*), are all included amongst "Things."

... enumeration of the

Or—

2. "So many highest predicates, one or other of which was supposed capable of being affirmed with truth of everything whatever."]

The Aristotelic Categories are:—

(1.) *Substantia.*	(2.) *Quantitas.*
(3.) *Qualitas.*	(4.) *Relatio.*
(5.) *Actio.*	(6.) *Passio.*
(7.) *Ubi.*	(8.) *Quando.*
(9.) *Situs.*	(10.) *Habitus.*

("*Habitus*," according to Sir William Hamilton, expresses the relation of the container to the contained; the idea being that of a man contained in his garments.)

Mill's criticism of the Aristotelic Categories:—

1. *The list is unphilosophical and superficial;* being a mere catalogue of the distinctions rudely marked out by the language of familiar life, without any attempt to penetrate to the rationale of even these common distinctions.
2. *It is Redundant;* *Actio, Passio, Ubi, Quando, Situs,* and *Habitus* are cases of Relation; *Situs* and *Ubi* are the same, viz., position in space.
3. *It is Defective;* having no head, or *summum genus,* under which *States of Consciousness* can be classed.

(It may be remarked, however, that good authorities maintain that Aristotle's classification was mainly *grammatical,* and was not intended by him as a list of the Categories in the usual *Logical* sense of the word.

"*Ubi;*" and the different forms "*Habitus,*" "*Actio,*" "*Passio.*" distinctions in the grammar of w this arrangement; and its appear may be accounted for by rememl day there was not that sharp line Logic, Metaphysics, and Grammar

II. Regarding the Categor enumeration of the classes of Mr Mill gives first a *Prelimin* of such classes; and, afterwar ing and giving a more philo this list, he constructs his *Fi* *Enumeration* of the Categories

1. *Preliminary list of Classes*

I. *Feelings,* or States of Consciousness or of Mind. — Se

II. *Substances.* — Bodies — Occ k c; Minds — Th

III. *Attributes.* — Quality. Quantity. Relation.

Respecting this enumeration it is only necessary to remark—(1.) That "*Substance*" here means anything which possesses attributes. *Mind* possesses attributes, properties, or qualities; hence it is a "Substance" in this sense. (2.) That Class IV., though merely including peculiar cases of Relation, is yet better placed apart, because the relations therein contained are peculiar, irresolvable and inexplicable. We know what we mean when we say one thing follows another, one thing coexists with another, or one thing is like another (we are speaking of *ultimate* resemblance),—we know what the state of mind is which accompanies the recognition of "Sequence," &c.; but we cannot pretend to explain or analyse it.

Mr Mill, however, goes on to show that *Attributes* are resolvable into *States of Consciousness*, and by including them in that Category, and treating "*Bodies*" and "*Minds*" as separate Categories, he arrives at the following *proper and final*

2. *Enumeration of the Categories:—*

I. *Feelings or States of* Consciousness (which includes *Attributes*).

II. *Minds—which* experience those Feelings.

III. *Bodies—external* objects which may excite certain *of those Feelings.*

IV. Co...

regarded as a Feeling? To explain this we may say that evidently *we know and can know nothing of Things, except through the Feelings they excite in us;* the Attributes, or properties, of things are, in fact, only other names for the powers which those things possess of exciting certain feelings. To say that an object before me is "*blue*," is to say that a certain mental state is excited in me—a certain feeling—which I call sensation of blue (quality). To say that an object possesses Attribute "*largeness*," is to say that it excites a certain sort of feeling in me; and so in every case of Attributes. That word properly means nothing more than the power of causing certain sensations in our minds,—and these sensations are all that, at bottom, we can mean by Attribute. To say that I am conscious of the Attribute or quality "*blueness*" in an object before me, is nothing more than to say that I am conscious of a certain feeling or mental state, called a sensation of blue. A similar analysis applies to Attributes of *quantity* and *relation* as well as of *quality*. The distinctions, therefore, which we verbally make between *the properties* of Things and *the sensations* we receive from them, must originate in the convenience of discourse, rather than in the real nature of what is denoted by those terms.

... Doctrine of Berkeley and the Idealist Meta-

it communicates by touch, and, in fact, the entire group of sensations which we can possibly have excited in us by it, to be similarly dealt with—what would then remain? Our ordinary conceptions would lead us to say the "thing itself," the "noumenon," the material substratum to which the properties or attributes belong. Berkely, on the other hand, would reply that nothing would be left,—that objects *are* nothing more than a bundle of sensations bound together by a fixed law; and that a fixed law of connexion making the sensations occur together, does not necessarily involve a material substratum. Grant such a substratum to exist, and, in an object before us, to be instantaneously annihilated by Almighty fiat, the sensations being, however, preserved unchanged, and how should we miss the supposed substratum? Evidently we should know nothing of its absence,—the object would remain *absolutely* the same *to us.*

Mill's remarks upon.—There is at least this much truth in this doctrine, *that* all we can know of objects is the *sensations* which they give us, and the order and *connexion of* those sensations. There is not the *slightest reason* for believing that the sensible *qualities manifested* by a thing are a type of anything *inherent in the thing* itself. An effect need not of *necessity resemble* its cause; the sensation of cold *is not similar* to anything ...

CHAPTER IV.

On Propositions.

A Proposition is a sentence in which a predicate is affirmed or denied of a subject.

Nature and office of the Copula.—The Copula is the mere sign of predication—the sign of *the connexion* between Subject and Predicate.

It does *not* indicate the actual existence of the subject; the notion that it does so arises from the double use of the verb "*to be*" (viz., as a mere sign of assertion, and as equivalent to "*to exist*").

[Since the copula indicates the connexion between the two terms, whatever has to do with that *connexion*, rather than with either of the terms separately, may be regarded as belonging to the copula. See *Negation* and *Modality*.]

Mill discusses the following distinctions in Propositions:—

I. Affirmative and Negative.

I. *Affirmative Propositions* are those in which the predicate is *affirmed* of the subject.

Negative Propositions, those in which a predicate is *denied* of the subject.

Hobbes' Theory of Negation.

Hobbes and some others state this distinction differently; they recognise only the *affirmative* copula (*is*, *are*, &c.), and attach the negative *to the predicate* (thus, *man is not-mortal*), and this with the idea of simplifying, by getting rid of, the distinction between affirming and denying, by treating every case of denying as the affirmation of a negative name.

Mill's remarks on.—The distinction between affirming and denying is real, and is not to be got rid of by a verbal juggle. A negative name is merely one expressive of *the absence* of an attribute,—so that when we affirm a negative name, we really affirm the *absence*, not the *presence*, of anything; not that something *is*, but that it is *not*. To put things together, and to put or keep them apart, will remain distinct *operations* whatever tricks we may play with language.

Modality, like Negation, belongs to the Copula: is not that ...

many different ways in which "sun" and "rising" are connected. If a doubtful case should arise, as to whether a given part of a Proposition belongs to Copula or to Predicate, ask the question—Does it modify the meaning of predicate, or does it rather affect the mode or kind of connexion between the subject and predicate?

Propositions which merely assert a state of mind relative to a fact, and not anything directly in the fact itself, form a special class. They do not assert a connexion between two things, but what we think about that connexion. Thus, "Cæsar may be dead," "C. is, perhaps, or probably dead," and such like, equal "I am not sure C. is alive," or "I am not sure that he is dead."

II. *Simple and Complex Propositions.*

A *Simple Proposition* is one in which one predicate is affirmed or denied of one subject.

A *Complex Proposition* has a plurality of subjects, or of predicates, or of both. "*Conditionals*" and "*Disjunctives*" are the most important of this class.

cal proposition (as far as there is any distinction) is :—that a conditional proposition is a proposition concerning *a proposition;* the subject of the assertion is itself an assertion.

This property, even, is not peculiar to conditional propositions; there are other sorts of assertions which may be made concerning a proposition, besides its inferribility from something else. Like other things a proposition has attributes which may be predicated of it; one of these attributes is its *inferribility* from another statement, as just said; but many others frequently occur. Thus, "C is D is an axiom of mathematics;" "C is D is a truth of Scripture, and a tenet of Protestants," &c. &c.

The important *position which conditional propositions hold in logical treatises* is simply owing to this,—that what *they* predicate of a proposition, its being an inference from something else, is precisely that one of the attributes of the proposition with which Logic is most intimately concerned.

[To prevent confusion, notice that Mill adopts Whately's term "*Hypothetical*," as a *genus* including both "*Conditionals* proper," and "*Disjunctives*,"—since Disjunctives are resolvable into Conditionals.]

Disjunctive Propositions :—

Any one of these may be resolved into two or more *conditionals*, thus—

"*Either A is B, or C is D*," is equal to
"*If A is not B, C is D*," } taken together.
"*If C is not D, A is B*,"

A Categ...

III. *Universal Propositions, &c.*

A general name is said to be *distributed* when it stands for each and every individual to which it can be applied as a name; or, shortly, when it stands for the whole of its denotation.

A *Universal Proposition* is one whose subject is *distributed.*

A *Particular Proposition* is one whose subject is *undistributed.*

A *Singular Proposition* has for its subject an individual name.

(Not necessarily a *Proper Name*, but often one of those *connotative or descriptive individual names* which have been already mentioned. As, "The Founder of Christianity was crucified.")

An *Indefinite Proposition* is one in which we know not whether it is Universal or Particular. They are only indefinite *in form*, the framer must know whether he meant it to be universal or not.

CHAPTER V.

The Import of Propositions.

The main question discussed in this chapter is this,—What ...

Since a Proposition consists of the Subject and Predicate, connected by the Copula, the main question naturally subdivides into two (as above), which will be discussed under I. and II., while under III. some miscellaneous points will be examined. These two questions are:—

I. Between what is a connexion asserted in a Proposition? in other words, What must we understand its terms to stand for or represent?

II. Having settled what the Subject and Predicate represent, we next inquire, What kind of connexion between them may be asserted?

I. What is it of which we really speak in an assertion? What do the terms of a Proposition stand for?

In considering this question, Mill first examines *three* views which have been advocated by different logicians; and, after criticising these, he gives his own doctrine.

1st Doctrine.—*That* a Proposition is the expression of a connexion between two *Ideas* (*i.e.*, that the terms *must be* understood to represent the corresponding *ideas*).

That is,—*that a Proposition* affirms or denies one *idea* of *another; that* "judging" is putting two ideas ... *bringin*...

these operations. Thus, "Gold is yellow"—these logicians would say means that "My idea of gold includes or agrees with my idea of yellow."

Mill's Criticism:—

This doctrine is a serious mistake. It is true, as before stated, that an idea or notion of the things brought into connexion is a necessary preliminary to, or *condition of*, the intelligent assertion of a Proposition ; but the assertion itself refers not to the ideas, but to the external facts.

2d Doctrine.—That a Proposition expresses the relation between *the application of two names.*

This was the doctrine of Hobbes: "In every Proposition," says he, "the thing signified is the belief of the speaker that the predicate is a name of the same thing or things of which the subject is a name;" in other words, a Proposition *merely* asserts that the two names which compose the terms have or have not been assigned to the same object or objects. Thus, "Man is a living creature," means that "living creature" is a name of everything of which "man" is a name; and if so, the Proposition is true; if not, false.

Mill's Criticism.—It may be remarked of this:—

... it is a property which all true Propositions ...

account are that very unimportant class where both terms are proper names—as, "Tully is Cicero."

(4.) Of all other Propositions it is a very imperfect analysis. Hobbes himself allows that general names are given to things, not accidentally, as it were, but because those things possess certain attributes, and it is strange that he did see that when we predicate of any subject a name given because of the possession of certain attributes, that our object is not to affirm *the name*, but *by means of* the name to affirm the *attributes*.

3d Doctrine.—That the terms represent *classes*.

That is, that an assertion consists in including an individual or a class in another class, or excluding the one or the other from it. Thus, "Plato is a philosopher," means that Plato is included in the class "philosopher;" "all men are mortal," that the class "men" is included in class "mortal beings." This theory is, in fact, framed as if nature had arranged all the objects of the universe into definite *à priori* classes; things of the same kind being, as it were, done up together in parcels or bundles, called classes, so that it is sufficient for us to know in what parcel or class any given object is included, and we have then all the information as to its attributes which we can require.

Mill's Criticisms:—

the human mind; observing that certain objects have certain properties in common, we group them together into a class in virtue of these common attributes, and give them a name which connotes these attributes. The error in the theory, then, consists in this—objects are (in real fact) first put together into a class because they possess certain attributes, and then (according to this view) those attributes are inferred because the objects belong to the class.

(2.) This theory is further essentially the same as that of Hobbes; to refer anything to a class is the same as saying the class name is applicable to it; for both the name and the class are dependent upon the possession by the objects of certain attributes.

(3.) This theory is the basis of the famous "*Dictum de omni et nullo*;" the syllogism being resolved into an inference that whatever is true of a class is true of everything contained in that class.

4th Doctrine.—Mill's view.—That a Proposition asserts the connexion between *what is connoted* by the terms.

That is, that every *connotative* term must be understood to stand for what it connotes; the *connoted attributes* are the things between which in a Proposition the connexion is asserted. A *non-connotative* term stands, of course, for its denotation,—*i.e.*, the objects to which it is applicable.

Instead of saying "attributes connoted by," we may substitute "phenomenon" without altering the meaning.

II. We now proceed to the second of the two questions,—as to what kinds of connexion may be asserted in Propositions.

The matter of fact asserted in every (real) proposition is one or other of these :—

Simple Existence.
Coexistence { the fact of = Order in Time. / the mode of = Order in Place.
Sequence.
Causation.
Resemblance.

But first—*Causation* is only a case of *Sequence*, which is itself a case of *Order in Time*. Again, *Coexistence* is either *Order in Time*, that is Simultaneousness; or *Order in Place*, *i.e.*, arrangement, collocation; as, "The boys stand in a straight row." We, therefore, arrive at the following final arrangement :—

1. Simple Existence.
2. Order in Time.
3. Order in Place.
4. Resemblance.

1. Propositions asserting or denying *the actual existence* of something require no particular remark. They are such as—"There are such things as black " "White crows do not exist."

2 & 3. Propositions asserting the *Conjunction* (which includes Order in Time and in Place) in some way of two Phenomena, especially those which affirm or deny Coexistence or Sequence, are of great importance in Logic.

Almost all Propositions expressive of physical facts, assert either the coexistence of two phenomena, or the sequence of one phenomenon after another. All Propositions asserting that one phenomenon is *the cause* of another, evidently assert sequence. The most important Propositions which assert *Order in Place* are mathematical loci; that is, Propositions which affirm that a succession of points marked in a certain way will lie along a certain path.

Propositions asserting *Order in Time* (Simultaneousness or Coexistence in Time, and Sequence) are so important that, for the general purposes of Logic, they may be taken exclusively. Mill does this; afterwards considering separately those which assert *Existence*, and those which assert *Resemblance*, and *Order in Place*.

It is evident that if one phenomenon is always *conjoined* with another, whether as simultaneous or as successive, that phenomenon becomes *a mark of* the other, *i.e.*, whenever we meet with the one we may be sure of the presence of the other; and every proposition of conjunction comes within this formula—"*One phenomenon is a mark of another phenomenon.*"

4. Propositions asserting *Resemblance* (or *Dissimilarity*).

mate resemblance, where two things are like each other, but we cannot *analyse* that likeness,—it is simple, and incapable of being resolved into more elementary particulars; and (2.) *Resemblance in some assignable respects.* Thus, if I say "This horse resembles other horses," I can enumerate the details of form, colour, &c., &c., in which the animals agree; but if I say this "Red is the same as that red," or "The hunger I feel to-day is like the hunger I felt yesterday," the resemblance cannot be thus analysed; we can only say that there is a resemblance, we cannot say *in what it consists.* It is evident, however, that any resemblance of the second kind between two things must consist of an aggregate of resemblances of the first kind. The *ultimate resemblances* are resemblances in our *simple feelings* (*i.e.*, conscious mental states not compounded of more elementary states).

The most important Propositions asserting Resemblance are those of Mathematics; resemblance having then the form of equality or proportionality.

III. *Certain Miscellaneous Questions.*

1. ***Propositions having a general name as predicate do not properly assert resemblance.***

The contrary has been maintained by some; they would say the assertion, "Gold is a metal," means "Gold resembles metals more than it does any other class of bodies." But a class is usually not founded upon a mere general unanalysable resemblance, but upon resemblance in certain assignable attributes common to all its members. These attributes are connoted by the class name, and it is the possession of these by the subject which we mean to assert, and

not mere general similarity. It might still be said, "Gold is a metal," *i.e.*, "the attributes connoted by name gold are accompanied by attributes connoted by name metal," if no other metal existed; just so we may say, "The Lord is God," though there is none other beside Him.

There are, however, two exceptional cases, i.e., ***cases where mere resemblance is predicated by a general name:—***

(*a.*) Where an individual is put into a class simply because it resembles the members of that class more nearly than of any other, and not, as usual, because it possesses the distinctive attributes of that class. Thus we say, "*Arsenic is a metal,*" though it differs in many respects from the other metals, because it resembles them more nearly than it does any other class of the elements.

(*b.*) Where the class corresponding to predicate name consists of members which have *only an ultimate resemblance*, and not a resemblance in specific assignable particulars.

The classes in question are those into which our *simple feelings* may be divided—"white," "red," "bitter," "sweet," &c. If I say "This tastes bitter," at bottom I *only* mean that it resembles other tastes which I have previously known under that name, and it would not be really understood by any one who had never experienced a bitter taste.

2. *Propositions whose terms are abstract* require no separate remark. They may always easily be changed into Propositions with corresponding concrete terms. The abstract name *denotes* the attri-

butes which the concrete *connotes*; "humanity is a mark of mortality," = "man is a mortal being."

3. *Negative and Particular Propositions* only require a slight corresponding alteration in the expression, thus:—

"*No horses are web-footed*," = "the attributes connoted by horse are a mark of the absence of those connoted by web-footed."

"*Some birds are web-footed*," = "attributes connoted by bird are sometimes accompanied by attributes connoted by web-footed."

It may be here remarked, once for all, that we consider *the absence of an attribute* as being itself an attribute. Thus, *e.g.*, we call "*not blue*," an attribute. It is a convenience which avoids circumlocution.

CHAPTER VI.

VERBAL AND REAL PROPOSITIONS.

Verbal Propositions (= Essential = "Analytic" of Kant) are those in which the connotation of Predicate is part or whole of the connotation of the Subject.

To these we may add those Propositions in which both predicate and subject are proper names.

A Verbal Proposition asserts, therefore, of a thing only what has already been implied when we uttered the name of the thing. To say "Man is an animal" conveys no information, "animal" being part of the very meaning of the word "man;" for we should certainly not call anything not animal "man."

Verbal Propositions, therefore, do not properly assert *matters of fact*, but only inform us as to the *meaning of names*.

Real Propositions (= Accidental = "Synthetic" of Kant) are those in which connotation of Predicate forms no part of connotation of Subject.

All Propositions, therefore, which have a proper name for the subject (and not for predicate also)—since such names connote nothing—come under this class. If the subject be a general name, it is, as already said, necessary that its connotation should not include that of predicate. Thus—"Man is a being which cooks its food;" "cooking his food" is not included in the meaning of "man."

II. *Assumption in Propositions of the Real Existence of the Subject.*

Take this Proposition "A is B"—under what circumstances are we to understand from the Proposition itself that A actually exists?

1. *Verbal Propositions* do not, *in strictness*, imply that the subject really exists; they are, as we shall see really more or less perfect *definitions* unfolding the connotation or meaning of a name; and for the ordinary copula "*is*" we may substitute "*means*,"

without altering the assertion. This is clearly seen where the subject connotes a group of attributes brought together by the imagination only; thus—"A centaur is a being half man half horse,"="A centaur means a being," &c.

Nevertheless, *as a matter of fact*, most Verbal Propositions do involve a tacit assertion of the real existence of the subject. In such cases *the apparently simple assertion really consists of a definition together with a postulate;* thus—"A triangle is a three-sided figure," = "Triangle means three-sided figure,"+"Such a figure exists;" the last being the postulate or assertion of actual existence. The importance of this will be seen hereafter.

2. *Real Propositions* do necessarily imply the real existence of the subject, because if the subject be non-existent, there is nothing for such a proposition to assert.

Apparent exceptions may occur to this—thus we may say, "*A ghost haunted his bedroom,*" "*The gods dwell on Olympus;*" but even here it is evident that the actual existence of "*ghost*" and "*gods*" is *pro tempore* assumed.

III. *Uses of Verbal Propositions.*

1. *With* the assumption or postulate of the actual existence of the subject:—
   - (*a.*) They may convey information of that existence as a fact.
   - (*b.*) They may serve as the basis of deductions, as do the axioms of mathematics.
2. *Without* that assumption—that is, in strict accuracy—they only serve as definitions,—*i.e.*, to unfold meaning or connotation of the subject.

With the exceptions here mentioned, *Real* Propositions are the only propositions which can convey any information as to *matters of fact*, or from which any *inferences* as to *matters of fact* can be drawn.

IV. *Two modes of stating import of a (Real) Proposition.*

(Mill here, as usual, takes Propositions asserting *conjunction* as *the* Propositions of Logic. See p. 31.)

(1.) Attributes connoted by subject *are accompanied by* attributes connoted by predicate; or

(2.) Attributes connoted by subject *are marks of*, &c.

These are really equivalent, but (1.) points to the Proposition regarded as a mere piece of knowledge; while (2.) points to its *practical* use.

CHAPTER VII.

I. On Classification; and II. The Predicables.

I. *Classification.*

(Classification as a scientific process is discussed in Book IV. Here only a few elementary points are noticed).

1 *Connexion of Naming and Classification.*

General names have a meaning quite irrespective of *classes* (that is, when we assert a general name, as "man," of a subject, we mean to assert that that

subject possesses certain attributes, "animality," "rationality," "upright form," &c., &c., which has nothing to do with classes), but there is a twofold *connexion* between them:—

(a.) *Classes mostly owe their existence, as classes, to general names.*

It is perfectly evident that if we invent a name, connoting certain attributes, we thereby *create* a class,—consisting of everything which possesses those attributes. Thus, suppose I take two attributes, "perfect molecular mobility" and "inelasticity," and devise a name "liquid," which shall connote or mean those properties, a class is *ipso facto* formed containing all objects possessing those two properties. Theoretically it matters not whether the objects are many, one, or none at all,—the class then being wholly imaginary.

(b.) *But sometimes, on the other hand, general names owe their existence to classes.*

A name is sometimes introduced because we have found it convenient to create a class. This is usually the case with the classes of Plants and Animals; as when we divide Plants into "Phænogamous" and "Cryptogamous," &c.

2. *Two kinds of Classes*— { *Real kinds.* / *Not-real kinds.* }

It is a fundamental principle in Logic that the power of framing classes is unlimited, as long as there is any, even the smallest, difference to found a distinction upon. Thus, out of the class "Man" we may cut a class "Christian," out of this a class "Protestant," again a class "Bishop," out of "Bishop" the class "black-haired," and so on *ad infinitum.*

But if we contemplate our classes when formed, we discover that they constitute *two* very broadly distinguished divisions,—*Real kinds and Not-real kinds.*

Real kinds are classes the members of which are characterised by the possession of an inexhaustible number of common properties, not referrible to any common cause.

Hence the *differences* between two individuals belonging to two distinct Real kinds are as innumerable as the *points of agreement* between two individuals belonging to the same Real kind.

Not-real kinds are classes the members of which agree only in certain particulars which can be numbered,—that is, which have only certain specific and determinate common properties.

Thus, compare the class "*Animals*" with the class "*White things;*" in the latter the members are not distinguished necessarily by any common properties except "*whiteness,*" and any properties effects of "whiteness;" but a hundred generations have not exhausted the properties which are common to all "*animals;*" and though physiologists are continually discovering new ones, yet there is no probability that they will ever be able to say that they know them all,—that they have arrived at a knowledge of every property which exists in "animals." Moreover, these common properties are not referrible to any one cause. "*Animals,*" therefore, is a Real kind; "*white things*" is not. Similarly in chemistry, the name of any element represents a class of objects. "*Sulphur,*" for instance, is a class including

every separate piece of sulphur. Will the time ever come when chemists can say that they have exhausted the whole list of the properties which those different pieces of sulphur possess in common? "*Sulphur*," like the other elements, therefore, is a Real kind.

It is sometimes said that *Real kinds are natural classes, or classes formed by nature*; we have already seen that classes are really framed by the human mind, but the expression is true thus far,—(1.) that Real kinds are classes for the recognition of which, as such, no elaborate process is generally required, because each of them is marked off from all others, not by some one or few properties which may be difficult to detect, but by its properties generally, by its *tout ensemble*; and, further, (2.) the ends of classification would be subverted if we did not recognise Real kinds as classes.

II. *The Predicables.*

The Predicables are a fivefold division of general names, to express the different kinds of class relation which may exist between the subject and predicate of Propositions.

Thus, take any Proposition, as "all A is B,"—the subject "A" represents a class or an individual, also predicate, being a general name, stands for a class "B," the Predicables express the different kinds of

The five Predicables are:—

Genus. Species.	Distinctions founded upon the Nature of Things, Genus and Species being Real kinds.
Differentia. Proprium. Accidens.	Distinctions founded on Connotation of Names.

A Genus is any Real kind which contains the subject, but which, at the same time, is divisible into *lower* (*i.e.*, less extensive) Real kinds, in one of which the subject is also contained.

Such a kind is a genus to all kinds below it; a species to all kinds above it.

"All kings are animals,"—here "animals" is *a genus* to "kings," since being a Real kind it includes a lower Real kind "man," to which also "king" belongs.

A Species (*i.e.*, an infima species) is the proximate or lowest Real kind to which the subject can be referred.

It is easily seen how these two distinctions are founded upon the nature of things, since whether a class is a Real kind or not, and whether it is the lowest Real kind in reference to a given subject, are clearly . . . the nature of things. Take

lars, they are distinct kinds. Now, when we say the "Hottentots are negroes," whether "negro" is the species to Hottentot depends upon whether "negro" is a Real kind or not,—a question of the actual facts of nature.

Differentia (in strictness) is the surplus of the connotation of the name of the species, over and above that of the name of the genus.

A species (as *man*) may be referred to a different *Genera* (*animal*, *vertebrate*, *mammal*, &c.), according to our purpose on the particular occasion. In any such case, it is evident that the name of the species will always connote all the attributes connoted by the name of the *Genus*, and also an excess of attributes peculiar to itself. Thus "man," besides connoting all that "mammal" connotes, connotes also a number of other attributes peculiar to itself (erect form, rationality, &c., &c.). It is this excess of connoted attributes which distinguishes "man" from all other species of *Genus* "mammal,"—in other words, forms its Differentia in respect of that *Genus*.

The Aristotelic Logicians, instead of taking *the whole* of such excess attributes, fixed upon *that one* of them which seemed to them the most important; thus, in the case of "man" they picked out "rationality" as *the* Differentia. Since this usually fulfils the purpose for which a Differentia is framed, *i.e.*, to *distinguish* the species, we may give a

Looser and more practical definition of Differentia, thus: The definition of a species is that part of the connotation of the specific name which distinguishes

the species in question from all other species of that particular genus to which we are referring it.

("That part" being composed of one or more, or all, excess attributes.)

[*Special or Technical connotation.*

1. In *names already connotative*, we may, if our purpose require it, change the connotation,—that is, we may select from amongst the common properties of the objects denoted by the name, a set distinct from those previously assigned to the name. Thus "man" ordinarily connotes animality, a certain form, and rationality; but in a natural history system it may be made to mean a certain arrangement of teeth, and the possession of two hands. This forms a *special* connotation of name "man."
2. Even *non-connotative names* may have a connotation assigned them in this way; thus, "whiteness" properly connotes nothing, but it may be made to mean "a mixture of the different tints of the spectrum," and similarly in other cases.]

A Proprium of a species is any attribute which, although not itself connoted by the specific name, yet follows from some attribute which that name does connote.

There are two classes of Propria:—

1. Those which follow by way of *consequence*, *i.e.*, as a conclusion from premisses.

   Thus, the attributes connoted by name "parallelogram" are—"having four straight sides," and "having opposite sides parallel;" *a proprium* (of

species "parallelogram"), "having opposite sides equal," follows by way of *consequence*.

2. Those which follow by way of *effect*, i.e., as an effect from a cause.

Thus, "man" connotes, amongst other things, the attribute "rationality," from this *a proprium* of "man" "capacity of using language" follows as an effect.

An Accidens is an attribute which, being neither connoted by the name of the species, nor following from any attribute so connoted, is yet found in the species.

Accidents are of two kinds :—

1. *Inseparable*—those which might, as far as we can see, be absent from the species without making it a different one, and yet never are so absent. As, "blackness" of species "crow."
2. *Separable*—those which are sometimes present, sometimes absent in individuals of the species. As, "red-haired" of species "man."

A fortiori, these accidents are "separable" which are not *even constant* in the individual, as "being clothed," "being ill," &c.

CHAPTER VIII.

On Definition.

A Definition is a Proposition declaratory of the meaning of a word; or, more precisely, it is the statement in words of the constituent

parts of the facts or phenomena of which the meaning of every word is ultimately composed.

Place in Logic and Use.—Definition is the Logical instrument of the first division of the Science, that relating to *Terms*. It remedies their indistinctness, by giving a precise and fixed meaning to every name capable of having such a meaning assigned to it, so that we may know precisely what attributes it connotes, and what objects it denotes. It is only in this way that our assertions can have a fixed and determinate import.

Definitions are either :—

I. *Perfect.*

II. *Imperfect.* { Incomplete Definitions. Accidental Definitions.

Perfect (=Complete, Adequate, Scientific) *Definitions* are those which declare *the whole* of the facts which the name involves in its signification; that is, in the case of connotative names those which unfold the whole connotation.

Imperfect Definitions are those which do not do this.

(These distinctions apply more or less to the definition of every name, but since the only *non-connotative names* which can be defined are the names of single attributes (see p. 12), which are defined by assigning the fundamentum of the attribute, this class

may be at once dismissed. Of *connotative names* (see also p. 12), the class of primary importance here is that of *concrete general names;* and this chapter may be considered as dealing with these exclusively, unless otherwise notified. All that it is necessary to say concerning *connotative abstract names* (names of groups of attributes, or of attributes which have attributes) will be given separately under that head).

I. *A Perfect or complete Definition* is one which expresses the whole connotation of the Name.

Form of a Definition.—The Definition must give the several attributes connoted by the name defined; now these attributes may either be enumerated *singly and seriatim*, or several may be grouped together under one word. In either case, again, we may name the attributes either directly by their own proper abstract names, or *indirectly* by using words which connote them. Thus, we get this arrangement:—

(1.) Attributes enumerated *singly*.	in either case	(*a.*) The attributes may be expressed *directly* by the abstract names which *denote* them; or
(2.) Attributes enumerated *in groups*.		(*b.*) The attributes may be expressed *indirectly*, by names which *connote* them.

II. *Imperfect Definitions* are either:—

1. Incomplete Definitions.	Name defined by *part only* of its connotation.
2. Accidental Definitions, or Descriptions.	Definition composed of some attribute which is *no part* of the connotation of the name defined.

Imperfect Definitions are framed with reference to one practical use of Definition,—the discriminating the things denoted by the name from all other things,—rather than with regard to scientific accuracy.

1. *Incomplete Definitions* serve the practical purpose of Definitions when it happens that all objects which possess the enumerated attributes possess those also which are omitted.

Logical Rule that Definition should be "per genus et differentiam."

Incomplete Definitions seem to have been had in view by Logicians when they laid down this well-known rule. It may be observed of it:—

1. That it would be better expressed "*per genus et differentias*," as it might then yield a complete Definition.
2. It is impossible thus to define all names capable of being defined,—*summa genera*, for example.
3. The object aimed at by those who laid down this rule is unattainable; they seemed to imagine that the function of Definition is to expound the division of

things into Real kinds, and to show the position which each kind holds in reference to other Real kinds; but this is impossible.

2. *Accidental Definitions or Descriptions.*

Are definitions composed of any attribute or combination of attributes which (though they are not connoted by the name defined) happen to be common to the whole of the subject, and peculiar to it.

Thus :—"Man is a self-clothing animal."
"Man is a food-cooking mammal."

It is only necessary to a Definition of this kind that it should be convertible with the name it professes to define; that is, it should be predicable of everything of which the subject is predicable, and of nothing else.

[*Special or Technical Definitions.*

Such accidental definitions may be raised to the rank of incomplete or even of complete Definitions, by *making* the elements of the description part or all of the connotation of the name defined.

Thus :—"Man is a two-handed mammal" is Cuvier's definition of "man,"—what with him the name "man" actually connotes.]

III. *Definition of Abstract Names.*

1. *Connotative Abstract names* (viz., names of groups of attributes, or of attributes which have attributes) may be defined like concrete names, by enumerating the attributes which they connote; the definition in fact being parallel to that of the corresponding concrete terms.

Thus :—Humanity = Corporeity, Animality, rationality, erect form.
A human being = A corporeal, animated, rational, erect being.

2. *Non-connotative abstract names* (*i.e.*, names of a single attribute) must be defined by *analysing the fundamentum* of that attribute; *i.e.*, by enumerating the facts or phenomena which the attribute represents.

Thus :—Eloquence = the faculty of influencing the affections of men by means of language.

IV. *What Names can and cannot be defined.*

1. *Every name whose meaning can be analysed can be defined,*—whether concrete or abstract; that is, every name in reference to which we can distinguish into parts, the attributes or set of attributes which form its signification.

[Even when the fact or phenomenon is one of our simple feelings, and, therefore, incapable of analysis, the names both of the *object* which excites, and of its attribute or property of exciting the feeling, may be defined by saying that they do so, but the name of the feeling itself cannot be defined. Thus :—

White thing = an object which excites the sensation of white.
Whiteness = property of exciting the sensation of white.]

2. *The names which cannot be defined are* :—
   1. Proper names—since they have no meaning.
   2. The names of our simple feelings, because their meaning cannot be analysed.

Thus the names "sensation of white," "sensation of pain," of "weariness," of "hunger," &c., &c., only mean similarity to sensations we have previously been accustomed to call by those names; and if we wished to convey a notion of them to another, we could only do so by calling up something similar in his own experience. Language is not adequate to explain colour to a man born blind; the similarity to previous feelings which the names of sensation of colour connote not being appreciable by him.

V. *Doctrine that Definitions are either of Names or of Things.*

Many Logicians have divided Definitions into two classes, thus :—

1. *Nominal Definition*—Definition of a name, *i.e.*, explaining the meaning of a name.
2. *Real Definition* —Definition of a thing, *i.e.*, explaining the nature of a thing.

In truth, all Definitions are definitions of *names*, and of names only, but what they confusedly perceived, and therefore vaguely indicated, is really the distinction, not between definitions as such, but between *definitions without, and definitions with, a postulate or assumption of the real existence of things corresponding to the name defined.* (See p. 36).

1. Definitions which *merely* declare the meaning of a name, without any assumption as to real existence of the subject. This they called "*Nominal Definition*," or definition *of a name*. Such declare nothing as to *matters of fact* properly so called, but only the

meaning which custom has assigned to a name, and therefore no conclusion as to matters of fact can be drawn from them.

2. Definitions which explain the meaning of a name, but at the same time assume the real existence of the subject. This they termed "*Real Definition*," or definition of *a thing*.

If in the former case we put "means" for "is," no change is made in the meaning of the proposition, nor is any inference which we may draw from it (being only inferences as to meaning of names) affected.

In the latter, the change leaves the true definition, but withdraws the implied assertion or postulate of real existence; and it is self-evident that no matter of fact could be inferred from a proposition which merely declares the sense in which we employ a word.

As *further arguments* in support of the view that *an apparent inference from a Definition is really an inference from the contained postulate,* Mr Mill

1. Examines Euclid I. 1, and shows that the very first step,—"about centre A with radius AB, describe a circle," depends upon the possibility of the *actual existence* of a circle;
2. And further shows that if the inference is really from the Definition, we may draw a false conclusion from true premisses—A dragon is a thing breathing flame, a dragon is a serpent: therefore, some serpent breathes flame. Here, in the major premiss, the definition is true, but the postulate is false, and the conclusion being false shows that our inference is really from the false postulate, not from the true definition.

[That definitions are really the premisses of scientific inference (as in Euclid) is sometimes defended by saying that they are so, provided that they are framed

conformably to the order of nature—that they assign such meanings to terms as shall suit actually existing objects. From the meaning of a name, we are told, it is possible to infer physical facts, provided the name has corresponding to it an existing thing. But from which, then, is the inference really drawn —from the existence of a thing having the properties, or from the existence of a name meaning them ?]

These postulates are not always exactly true,— the things whose existence they tacitly assert do not always exist exactly as defined.

Thus, probably no circle ever existed whose radii were *precisely* equal—no point without magnitude, no line without breadth, &c.

Hence these postulates are really suppositions or hypotheses; that is to say, we *suppose* such points, lines, circles, &c., to actually exist as described in the definitions, though they do not in fact.

This explains what Mill means by saying that "*geometry is based upon hypotheses or suppositions;*" which is evidently true, because we have already seen that its inferences (putting *the axioms* aside for the present) are really from the postulates in the definitions, and these postulates or tacit assertions are, in fact, suppositions.

Since mathematical truth (at least in part) rests ultimately upon such suppositions, a difficulty arose in understanding how the most certain of all truths could rest upon premisses which, so far from being exactly true, were certainly not true to the whole extent assumed. The real answer to this difficulty is, that *as much of the premisses is true as is required to support as much as is true of the conclusion.*

Some get over this apparent difficulty by saying that *the Definitions of Mathematics are really Definitions not of the things, but of our ideas of the things.* They say that what we argue about are our mental pictures or ideas of points, circles, lines, &c., which do *exactly* correspond to the Definitions.

On this Mill remarks :—

1. Even if we can form a mental picture of a mathematical point, line, &c., the definition postulates the real existence of such an ideal picture, and from those postulates all inferences must really be made. The case is in fact exactly the same as before, only we have a mental picture instead of one drawn on an external surface.
2. As a matter of fact, however, the mind can form no such pictures or notions as the hypothesis implies. We cannot conceive a line without breadth, &c. All that we can do is *to attend* to its length exclusively, neglecting the breadth for the time; and so in every case we can attend to and deal with a single element, as existing alone, but we cannot picture it to our minds as actually so existing.

Proper meaning of "Definition of a Thing."

In Book IV. Mill remarks that *if* we retain this expression in Logic, we must give it this meaning :—

The Definition of a thing, or rather of a class of things (for we cannot define an individual), is defining the name in such a manner that it shall still continue to denote those things.

VI. *An inquiry into the Definition of a Name* (*i.e.*, as to what the Name *should* mean)—

Is an inquiry into the specific attributes in which the objects which the name usually denotes, agree or differ. A definition is therefore not arbitrary, and often involves a long examination into the properties of things.

Thus, suppose we wish to define "*just actions*," it would be necessary to collect instances of actions to which that designation is applied, and to examine them carefully to determine in what they agree. Having settled the properties, qualities, or attributes common to all just actions, we have next to decide which of these are best adapted to enter into our Definition.

This inquiry is, however, rendered difficult by:—

1. The fact that words (particularly some abstract names) are constantly used without any definite connotation, except that of vague resemblance to other things called by the same name.

Thus, when ordinary people speak of an act as "just" or "noble," they often really mean nothing more definite than a vague feeling that the act in question resembles others which they have heard so called.

2. The transitive application of words. There is a constant tendency in men, when they meet with a new object, not to invent a new name for it, but to give it the name of some known object which seems to resemble it most. In this way a name may pass from object A to object B, from this to C, again to D, and so on till every vestige of definite signification is lost, and the various objects denoted by the name come to have really nothing in common. The name "beautiful" is perhaps an example of this process.

BOOK II.

INFERENCE OR REASONING IN GENERAL.

CHAPTER I.

Inference in General.

A Proposition is said to be proved when we believe it to be true by reason of some other fact or statement from which it is said to follow.

I. *Inferences improperly so called:*—

That is, the cases in which a Proposition, ostensibly inferred from another, appears on analysis to assert merely the very same fact, or part of the same fact asserted in the first.

1. *Æquipollency* or equivalence of propositions. Thus, "all men are mortal," for "no man is exempt from death."
2. *Sub-alternation*—All A is B ∴ Some A is B.
3. *Conversion*—As some sovereigns are tyrants ∴ some tyrants are sovereigns. Both propositions assert

the same fact—that the attributes connoted by "sovereign" are sometimes conjoined with those connoted by "tyrant."

4. *Repetition of connotation*—*i.e.*, when predicate of consequent is part of connotation of predicate of antecedent.

As "Socrates is a man," ∴ "Socrates is an animal." "Animal" being part of connotation or meaning of "man."

[5. *All other cases of so-called Immediate Inferences* may be added to Mill's list. Such as (1.) *Of Opposition*—A is B ∴ A is not non-B; (2.) *Of Relation*—A is the son of B ∴ B is father of A; and such like.]

II. *Logical forms of Inference.*

1. *Induction* is reasoning from particulars to generals; or, more correctly, inferring a Proposition from Propositions less general than itself,—the conclusion being more general than the largest of the premisses.

2. *Ratiocination* (= Syllogism) is reasoning from generals to particulars; or, more correctly, inferring a Proposition from Propositions equally or more general than itself,—the conclusion being less or only equally general with the largest of the premisses.

3. *Inference from particulars to particulars*—that is, from individual cases to another individual case.

If from having seen A, B, C, and some other persons die (supposing, of course, that I know nothing except what I observe myself), I infer E also is mortal, I evidently reason from the former particular cases to

the yet untried case. A little consideration will show us that a large number of our every-day inferences are of this sort.

There is, however, no real difference between this mode of reasoning and Induction. For it is clear that, in order to be logically warranted in my conclusion that E is mortal, I must have evidence enough from my antecedent particular cases to support the inference "*Any* man is mortal,"—for if the evidence did not amount to proving this, how could I be sure that E might not be amongst the exceptions? But if it prove "any man is mortal," it of course proves "all men are mortal,"—*i.e.*, it is *an Induction*, or the inference of a general proposition from particular cases. As Mill says—"Whenever we are *logically* warranted in arguing from a set of particular cases to some new case, we are also warranted in inferring the *general* proposition, which includes that new case," and this being so, we may consider this mode of reasoning as identical with Induction.

Further, then, since (as will be shown hereafter) Ratiocination is only *the interpretation and application of Inductions*, all Logical Inference consists in Induction, or in the interpretation and application of Propositions arrived at by Induction.

Induction is without doubt a true process of inference—the facts stated in conclusion are *bonâ fide* different from the facts given in the premisses. If, from examining four cases of mortality in man, I infer E also is mortal,—this last fact is clearly distinct from the others; still more if I lay it down that "all men are mortal," I assert innumerable separate facts,—X is mortal, Y is mortal, &c., &c.

CHAPTER II.

The Syllogism.

In the Analysis of the Syllogism Mill takes the two elementary forms of the first figure, *Barbara, Celarent*, as the universal types of all correct Ratiocination,—the first for affirmative, the second for negative conclusions. An argument in any other figure may be reduced to one of these.

I. *Initial Analysis of the Syllogism.*

In both these general types the major premiss is universal; all Ratiocination, therefore, starts from a *general* Proposition.

The minor premiss is affirmative, asserting that something is contained in the class of which something has been affirmed or denied in the major premiss.

The conclusion, then, infers that what is affirmed or denied of the entire class may be affirmed or denied of the objects asserted to be in that class.

It was by regarding this analysis as sufficient and exhaustive, that the *Dictum de omni et nullo* came to be accepted as the Axiom of Ratiocination.

II. *This dictum is a mere identical Proposition, and not the fundamental Axiom of Syllogism.*

A (whole) class is, in fact, the same thing as (all) the individuals included in it. Therefore if we say—

"Whatever is true of a whole class is true of every individual in that class," it is the same as saying—"Whatever is true of all the individuals of a class is true of every individual in it,"—an obviously identical Proposition.

To give, therefore, any meaning at all to the Dictum, we must regard it, not as an *axiom*, but as a roundabout *definition* of a class (*i.e.*, an assemblage of individuals of which the same Proposition is true).

III. *Fundamental Axiom of Ratiocination.*

"Whatever (A) is a mark of any mark (B) is a mark of that (C) which this last (B) is a mark of."

That is—if A is a mark of B, and B a mark of C, then A is a mark of C.

Another form of the Axiom of Syllogism is—

"Whatever possesses any mark, possesses that which it is a mark of."

That is—A possesses B, which is a mark of C, therefore A possesses C. This is really identical with the first, but a little varied in the expression. Either may be taken.

CHAPTER III.

FUNCTIONS AND VALUE OF SYLLOGISM.

I. *Does the Syllogism really involve a* Petitio Principii ?

(This question must be understood to mean—Does the conclusion of a Syllogism assert any *fact* or *facts bonâ fide* new, and distinct from what has been asserted in major premiss ?)

"It must be granted," says Mill, "that in every Syllogism, considered as an argument to prove the conclusion, there is such a *Petitio Principii*," *i.e.*, no really new fact is asserted in conclusion. This will be clearly shown by the analysis of the Syllogism itself, but the following is a summary of the arguments which bear on the question :—

1. That the Syllogism is conclusive from its *mere form*, *i.e.*, by a comparison of the language. Clearly this could not be the case if any new *fact* were asserted in conclusion.
2. The accounts given by its defenders :—
   (*a.*) That it is vicious if the conclusion assert anything *more* than is asserted in the premisses.
   (*b.*) That its function is merely to prevent inconsistency in our opinions.
3. An examination of the Syllogism itself shows that

both major premiss and conclusion are ultimately inferences from the same set of particulars.

4. That the Syllogism is a process of interpretation is shown also by an examination of those cases where the major premiss is not derived from experience, as in Law and Theology.

II. *What, then, is the true nature of the Syllogistic process ?*

The following familiar explanation will, perhaps, give a clear view of this important point :—

Suppose we have observed A, B, C, and D to be mortal ; let us assume that these particular cases constitute evidence sufficient to justify us in concluding that any new case, X, is mortal ; this is so far a case of inference from particulars to particulars. But it has been already explained, that in order to be warranted in inferring that any indifferent individual, X, is mortal, we must be warranted in inferring "All men are mortal" (for if not, our X might be amongst the exceptions)—*i.e.*, any evidence which proves the particular Proposition must also prove the general. Thus :—

(1.) A, B, C, D, &c., are mortal { observed facts — the premisses or evidence.

(2.) All men are mortal (= attributes of man are a mark of attribute mortality).   (3.) X is mortal.

(2.) and (3.) being two conclusions, either of which may be drawn separately without thinking of the other ;

but if the evidence prove one, it must also prove the other.

But, further, instead of remembering the details of our evidence—the special particular cases from which our conclusion was drawn—we may, once for all, retain *a record or memorandum of all that it will prove* by simply bearing in mind the general Proposition. Having once satisfactorily shown that "the attributes of man are a mark of mortality" we may dismiss from our minds the antecedent particular cases which proved it, and remembering only the generalisation itself, may *apply* it to new instances as they arise from time to time,—the application consisting in the ascertaining that a new instance possesses the mark therein laid down (in the example—that the new object possesses the attributes of man, which are a mark of mortality). This process of applying a general proposition (the major premiss) constitutes the Syllogism; which is, in fact, a kind of reasoning in which, in place of the evidence itself, we substitute a record or memorandum of all that that evidence will prove, and then proceed to interpret and apply that record.

From this it is evident:—

1. That the conclusion is not really drawn *from* the major premiss, but *according to it. The record or summary of the kind of conclusions* we may draw from given evidence, must not be confounded with *that evidence itself.*
2. That *the Syllogism is not the mode in which we must reason, but only a mode in which we may reason.* We may, and, indeed, constantly do, infer from observed individual cases to a new case, without ever think-

ing of a general proposition—as from (1) to (3). If, however, we do pass through the generalisation, the whole process will stand thus:—

(1.) A, B, C, &c., are mortal ∴ (2.) All men are mortal	An Induction leading to a generalisation.
(2.) All men are mortal X is a man ∴ (3.) X is mortal	Interpretation and application of the generalisation = Syllogism or Deduction.

All inference, therefore, is fundamentally from particulars to particulars, with the option of passing through a general proposition: and *such a general proposition is a register or record of an inference already made, and a short formula for making more.*

III. *Advantages of throwing the result of an Induction into the form of a general Proposition:—*

1. *The Induction may be made once for all.* It is evident that having once proved the general Proposition, we need no longer trouble to remember the original particulars, but have only to apply the generalisation to new particular cases as they arise.
2. *A general Proposition brings before the mind all that our evidence will prove,* if it prove anything.
3. *It presents a larger object to the imagination—i.e.,* it is naturally felt to be of greater importance than any particular assertion, and hence—
4. *It tends to prevent us from being influenced by Bias or Negligence* in our Inference.
5. And, finally, *it brings before us all possible parallel cases.*

IV. *The use of the Syllogistic form, then, i.e.,* of passing through the general Proposition in our Reasoning,—is that it is a most important collateral security for the correctness of the Inductive process which gives us that generalisation. (See III. 2-5.)

V. *The use of the Syllogistic rules* is to secure accuracy in the interpretation of the general Proposition.

VI. *Syllogism the test of Reasoning.*

The *Syllogistic form* is a test of the accuracy of the generalisation; the *Syllogistic rules* are a test of the accuracy of the interpretation of the generalisation.

(If we substitute "*security for*" instead of "*test of*," perhaps the meaning will be clearer.)

VII. *Syllogism not the universal type of Reasoning.*

Because, as already shown, we *may* reason without passing through any general Proposition,—we *may* infer directly from observed particulars to a new particular case; and in simple and obvious cases we habitually do so. It is a matter of choice and convenience, therefore, whether we reason after the type of the Syllogism or not.

VIII. *What is the universal type of Reasoning?*

1. Certain individuals (having a certain attribute, A) have (also) a given attribute, B;
2. An individual or individuals resemble the former in possessing attribute A; analogy

∴ 3. They resemble them also in possessing given attribute B.

This type does not claim to be conclusive from the *mere form;* its validity in any particular instance must be determined by the canons of Induction.

IX. *Functions of Major and Minor Premisses in a Syllogism.*

1. The *major premiss* asserts something,—which has been found true of certain known cases,—to be true of all other cases resembling the former in certain given particulars. Or,

That one phenomenon is a mark of another phenomenon.

2. The *minor premiss* asserts that some new case resembles the former in the given particulars. Or,

That some new cases possess the mark asserted in the major.

[X. *Dr Thomas Brown's Theory of Syllogism.*

Dr B. dispensed with the major premiss, asserting that the premisses in a Syllogism consisted of the minor alone, thus:—

Socrates is a man;
∴ Socrates is mortal;

E

but allowing at the same time *the necessity of perceiving the connexion* between man and mortality, which is only another way of saying that we must have either actually or implied a major premiss, which asserts that connexion.]

XI. *Relation between Induction and Deduction.*

"Although, therefore, all processes of Inference in which the ultimate premisses are particular cases, whether we conclude from these particulars directly to a new case, or to a general proposition, *according to which* we afterwards conclude concerning new particulars, —are equally Induction, yet we shall consider

Induction as belonging more peculiarly to the first step, the establishing the general proposition—that one phenomenon is a mark of another phenomenon,—while

Deduction is the remaining operation, which is that of interpreting and applying that general proposition. And we shall further consider every process by which anything is inferred concerning a new case, as consisting of an Induction followed by a Deduction; because, though we *need* not carry on the process in this form, yet it *may* always be thrown into that form, and ought to be so when accuracy is essential."

XII. *Objections to Mill's Theory of Syllogism.*

1. *Whately's:—*

It cannot be correct to say that a Syllogism is only a

special mode of dealing with the conclusion of an Induction, for *the argument in every Induction is itself a Syllogism*. Thus, I observe that "A, B, C, &c., are mortal," and from this I infer "all men are mortal;" but in this Induction we tacitly assume that "what is true of A, B, C, &c., is true of all men," and our Inductive argument is in fact a Syllogism, of which this assumption is the major premiss.

Mill replies:—

There is no such major premiss in the Inductive argument as is here assumed; in fact, the proposition which Whately gives as such is more properly part of the *conclusion* of that argument. Whenever we draw a conclusion from evidence, we, of course, tacitly assert the sufficiency of that evidence to support the conclusion, and therefore an implied assertion of the sufficiency of the evidence may be said to be part of the conclusion. It is just so here, —the supposed major is really an assertion of the sufficiency of the evidence, and is therefore necessarily involved in the conclusion. The premisses of the Induction are simply and only—"A, B, C, &c., are mortal;" having observed these particular cases, we are led by an instinctive mental tendency to expect, at least, that the same will be true of the next case, and, finally, after what we think a sufficient number of observed cases, that it will be true of all men.

2. Others object that Mill's theory does away with the minor premiss.

"If, say they, the major includes the conclusion, then we

could affirm the conclusion without the minor, which we evidently cannot do."

Mill replies :—

When we say that the major premiss includes the conclusion, (that is, that the conclusion asserts the same facts, or part thereof, as the major premiss), we do not mean that it *individually specifies* all it includes. It only includes by giving marks; "all red-haired men are choleric," does not *name* the individuals, but it lays down a mark ("red-haired") by which we may know them, and we require a minor premiss to show us that a new case *is* included in the major by possessing the mark.

CHAPTER IV.

ON TRAINS OF REASONING AND DEDUCTIVE SCIENCES.

I. A *Train of Reasoning* (=the "Sorites" of other Logicians) is a series of Inductive Inferences, from particulars to particulars, through marks of marks.

[The formula of the ordinary Syllogism we have seen to be :—

A is a m... mark of (attribute) B (1...

X poss... sses (attribute)...

∴ X poss...

Here we are supposed to know our minor by direct observation—by seeing or feeling that X possesses A; and if so, to get the conclusion, we require but a single Induction, that which proves "A is a mark of B."

But it may happen that the fact that X possesses A is not obvious to the senses; direct observation, perhaps, only shows us that X possesses C, and we require a second Induction to prove that "C is a mark of A," when we get the required minor of first Syllogism by a second Syllogism. Thus :—

2. C is a mark of A (2d Induction).
X possesses C.
∴ X possesses A (minor of 1st).

But again it may happen that our senses do not inform us even that X possesses C; by direct observation we may only know that X possesses D; again we must go through a similar process; that "D is a mark of C" must be proved by a third Induction, and we must then frame a third Syllogism before the minor of our second can be proved, and, therefore, before our first conclusion can be logically established. Thus :—

3. D is a mark of C (3d Induction).
X possesses D.
∴ X pos...

It i...

C is a mark of A, A is a mark of B, ∴ X possesses B. If these Propositions be written down, each under the preceding, they will be seen to form the Sorites of ordinary Logicians.]

A more complex form of Trains of Reasoning is this type:—

E is a mark of D.
F „ of C.
G „ of B.
Also D C B is a mark of N.
∴ E F G is a mark of N.

II. *Why do Deductive Sciences exist?* or to phrase it more accurately—*Why are there any difficulties in Deductive Sciences?*

Sciences which employ Trains of Reasoning are Deductive Sciences; now each fresh step or Syllogism requires a separate Induction, and if these Inductions have been obtained, it would seem that very little difficulty would remain in merely framing the chain of Deduction. Yet we find very difficult Deductive Science may exist (as Mathematics) where the primary Inductions are of the simplest character.

The explanation of this is, that (1.) There may be much difficulty in finding an Induction which comprehends any given case; and (2.) It often requires the highest scientific ingenuity so to combine Inductions as at length to reach one which will directly include our case.

Mill illustrates this by proving Euclid I. 5, direct from

the six primary inductions which are necessary to that proof.

III. *Differences between Deductive and Experimental Sciences.*

The opposition is not between Deductive and Inductive but between Deductive and Experimental.

A Science is Experimental in proportion as every new case requires a new Induction,—*i.e.*, a new set of observations and experiments for its elucidation.

A Science is Deductive in proportion as every new case can be brought under an old Induction, *i.e.*, in proportion as every new case may be elucidated without a new set of observations or experiments.

The generic difference, then, between Deductive and Experimental Sciences is this: That in the former we have, in the latter we have not, been able to discover marks of marks.

IV. *How an Experimental tends to become a Deductive Science.*

Thus, in an Experimental Science, the Inductions lie detached—A is a mark of B; C is a mark of D; E of F, and so on. Now (1.) A new Induction may, at any time, connect two or more of these pairs by showing, for instance, that B is a mark of C, when we could infer without experiment that A is a mark of C, and even of D; and (2.) Some new Induction may connect hosts of these isolated pairs at once, thus making the Science largely Deductive at a single stroke. *It is, therefore, this class of discoveries which are most potent in changing a Science in this way.*

V. *The grand agent, however, for transforming Experimental into Deductive Sciences is the Science of Number.*

The properties of number are alone, in the most rigorous sense, properties of everything whatever. But these truths can only be affirmed of things in respect of their *quantity*. Now, if it come to be discovered that variations of *quantity*, in any class of phenomena, correspond regularly to variations of *quality*, every formula of Mathematics which relates to quantity becomes so far a mark of qualities in those phenomena, and thus so far renders the Science Deductive.

Thus, when in Geometry, it was shown that every variety in position of points, direction of lines, forms of curves, or of surfaces (all *qualities*) had a corresponding relation of size (quantity) between two or three co-ordinates, an unparalleled Deductive extension was thereby given to that Science.

CHAPTER V.

On Demonstration and Necessary Truth.

Necessary Truth, according to the common definition, is such as is supposed to be independent of the evidence of ...

stand the expression in this sense, but Mill would define it thus :—

Necessary Truth is such as necessarily follows from assumptions which, by the conditions of the inquiry, are not to be questioned.

What is really meant by necessity here, therefore, is *certainty of Inference.*

I. *The character of Necessity (and even, with some reservations, the peculiar certainty) attributed to Mathematical Truth is an illusion.*

As to their peculiar certainty, it only consists in this—that mathematical conclusions are not liable to be interfered with by counteracting causes. In *themselves* they are not more certain or exact than of any other science; for Induction being at the root of *all* science, the first principles of each, if validly inferred, must be equally exact; and if the Deductions are correct, the results must be equally certain.

The necessity, too, of geometrical truths consists only in this,—that they necessarily follow from the granting of the primary suppositions (or hypotheses) from which they are deduced. Their necessity, in fact, is certainly of inference.

These primary suppositions are :—

Some of these suppositions (viz., the hypothetical postulate in the definitions—which assume the actual existence of things corresponding to those definitions) are not only not necessary, they are not even strictly true. (See p. 52.)

Some others of the first principles of Geometry are axioms, which are absolutely true without any mixture of hypothesis or assumption. That "the whole is equal to the sum of its parts" involves no hypothesis of any kind.

II. *Axioms are Experimental Truths; Inductions from the evidence of our senses.*

That is, the evidence upon which we believe axioms is of the same kind as the evidence upon which we believe any other fact of external nature—our experience of their truth. They are, in fact, the simplest and easiest cases of generalisation from the facts furnished to us by our senses or imagination.

(In the following discussion Mill takes, for convenience, the particular axiom, "Two straight lines cannot enclose a space," or, "Two straight lines, having once intersected, continue to diverge, and never again meet." Most of the arguments are, however, applicable to any other axiom also).

moment the Proposition is understood, and without the necessity of verifying it by trial. "To learn a Proposition by experience, and to see it to be necessarily true, are two altogether different processes of thought."

In answer to this view, and in support of his own, Mill *first* shows that the evidence derived from experience is amply sufficient to prove axiomatic truths; and, *secondly*, examines certain arguments advanced against his theory.

1. *The evidence derived from experience is amply sufficient to prove axioms.*

Experience confirms them almost every moment of our lives; in the axiom we have taken, for example, we cannot look at any two intersecting lines without seeing that it is true. Experimental proofs crowd in upon us in such profusion, without a solitary instance of even the suspicion of an exception, that it is impossible to conceive more decisive proof from experience. Where, then, is the necessity of supposing any *other* evidence of the truth of axioms than that which is seen to be so amply sufficient?

2. *The following arguments have been advanced* ... *Mill's view*: they are given *seriatim* with

merely *thinking* of them, therefore the ground of our belief must be in the laws of the mind itself.

Mill replies :—

Imagination can so perfectly reproduce sensations of *form*, that our *mental pictures* of lines, circles, &c. are just as fit subjects of experiment as the *external pictures*, or the realities themselves.

(*b.*) The thing asserted being that the lines will *not meet* in infinity—how can the senses take cognisance of a non-existent phenomenon—can we see or feel the lines *not meet* at an infinite distance?

Mill replies :—

We know that if the two lines ever do meet, or even begin to approach, this must occur at some finite distance; and the perfection of our mental pictures of forms enables us to frame a mental image of the appearance of the lines at such a point—an appearance which would be inconsistent with our notions of a straight line.

(*c.*) The next and great argument is this :—

Not only are axioms conceived of as being truths, but as being *necessary* truths—truths which *could* not be otherwise; *Propositions of which the contradictory is distinctly inconceivable.*

Now, since experience can only inform us of what is, has been, or may be, and cannot possibly certify us of what *must* be, axioms cannot be based upon that evidence.

[This argument of Whewell's may be conveniently thrown into this technical form :—

The *necessary truth* of a Proposition is a mark of its *not being derived from experience* (experience cannot inform us what *must* be).

The *inconceivability of the contradictory* is the mark of the *necessary truth* of a Proposition;

∴ The *inconceivability of its contradictory* is a mark of a Proposition *not being derived from experience.*

Mill attacks and refutes the minor premiss as below.]

Mill replies.—Inconceivability of the contradictory of a Proposition is so far from being a mark of its (so-called) necessary truth, that it is not even a certain mark of its being true at all. Or, What is inconceivable is neither necessarily, nor always, false; for—

1. *Inconceivability is an accidental thing*, dependent on the mental constitution and history of the person who tries to frame the conception. Our capacity or incapacity of conceiving the truth of a Proposition depends chiefly on three things—(1.) The frequency and constancy with which we have found the Proposition true; (2.) Whether we have ever found it to be false; and (3.) If not, whether there exist any analogies which might suggest the possibility of its ever failing to be true. This is further proved and expanded in 2 and 3 following.

2. *We have several examples of Propositions, once regarded as inconceivable by the greatest men, now recognised not merely as conceivable, but as the only true accounts.*

   Thus, to Newton it was inconceivable that a body should act where it is not; yet now it is universally recognised in the theory of Gravitation, Magnetism, &c.

3. *Conversely we have examples of truths really arrived*

at by *long and complex investigations*, becoming so familiar, that by some scientific men they are held to be necessary truths, *i.e.*, truths whose contradictory is and must always have been inconceivable.

Thus, some have supposed *the first law of motion* to be a necessary truth in this sense; and also the doctrine of the uniformity of composition of chemical compounds. Thus we see that inconceivableness proves nothing, except that two ideas are so firmly associated in our minds that we find it impossible to disconnect them.

[*Sir Wm. Hamilton coincides with Mill in rejecting inconceivability as a certain mark of falsity*; to assert, he observes, that what is inconceivable is necessarily false, brings us into collision with the higher laws of thought; thus, matter must be either infinitely divisible or not, in virtue of the "Law of Excluded Middle," and it cannot be both of these, in virtue of the "Law of Contradiction." Now, either of these alternatives is inconceivable. We cannot imagine the subdivision of a material particle carried on infinitely, nor can we conceive a point at which that division must end, an atom so small that it could not be divided. But, as just said, by "Law of Excluded Middle," one of these inconceivable alternatives *must* be true; therefore something inconceivable is not necessarily false. Q.E.D.

(By the higher "Laws of Thought" Sir W. H. means chiefly these three axioms—"Law of Identity," A is A; "Law of Contradiction," A is not non-A; and "Law of Excluded Middle," A is either B or not B).]

[*These additional arguments*, though not noticed by Mill, may be thrown together here, because they are often given in support of the view that axioms are proved by an *à priori* law of the mind, and not by experience.

1. Increase of certainty, *pari passu*, with increased experience, is a mark of a truth derived from experience. For example, after seeing ten people die, I should expect more confidently the mortality of any new case than if I had seen five only, still more if I had seen one hundred than if merely ten, and so on, up to full certainty. Axioms want this mark, being believed, with *the fullest* certainty, *immediately* they are understood.

The reply seems to be—that after having once arrived at full certainty, no further experience can increase that certainty, and if, as happens in axioms and similar simple assertions, a single experience is sufficient to fully prove them, no further certainty *can* be given by increased experience, or anything else.

2. Impossibility of establishing a proposition by propositions simpler or more certain than itself, is a mark of the necessary truth of that proposition. Axioms possess this mark, and therefore are necessarily true.

3. Mankind universally, even those who dispute them in the abstract, constantly acting as if they believed them, is a mark of necessary truth. Axioms possess the mark.

4. There is to our minds a distinct and conscious difference between the two classes of truths, both as to their certainty (it seems quite impossible to get rid of the belief) and the kind of evidence we should mentally fall back upon if they were disputed. Compare in this respect the proposition, "Two and three make five," with the assertion "Fire burns,"—

the former seems to have a necessity about it which does not belong to the latter.

The reply is simply—that two and three are just as true of propositions which are admitted to be proved by experience, and therefore are *not* marks of (so-called) necessary truth; while four only asserts the inconceivableness of the contradictory in different words.]

III. *Herbert Spencer's Doctrine of the Universal Postulate.*

Mr Herbert Spencer, while agreeing with Mill that the inconceivableness of the contradictory of a Proposition is not always a mark of its truth, yet maintains the view that this inconceivability of the contradictory is really the basis of our belief of axioms, because it is *the ultimate basis of all our beliefs.* This is his reason for terming it the universal postulate. He lays it down that, whenever a Proposition is invariably believed (that is, by all men always), it is true; and the mark of its being invariably believed is the inconceivability of its contradictory, so that we may phrase the universal postulate thus:—

Whenever the contradictory of a Proposition is inconceivable, that Proposition must be accepted as true.

This, he says, is really assumed in all our beliefs; if they are *intuitive*, as when we assert that we taste a bitter taste, the real ground of our belief that we are experiencing it, is the impossibility of conceiving (*i.e.*, believing) the contradictory—that we are not. So in belief of *inferences*, we believe that the conclusion follows from the premisses, because we cannot conceive it not following. Being thus the final ground of *all* our beliefs, it must be that of axioms amongst the rest.

This view is unfolded in and supported by these two arguments:—

(1.) Invariableness of belief in a Proposition (of which inconceivability of contradictory is the mark) represents the sum or aggregate of all past experience. Facts of every kind are continually coming before us, and impressing themselves upon us; our experience is a register of such facts, and the inconceivableness of a belief shows that it is altogether at variance with that register.

To this Mill replies.—(1.) Even if inconceivableness represent the net result of all past experience, why not appeal to that experience itself, and not presume it from a mere incidental consequence? (2.) But uniform experience is by no means an unfailing criterion of certain truth; and (3.) Not only is uniformity of past experience far from being a test of certain truth, but inconceivableness is very far from being a test even of that imperfect test. Uniformity of contrary experience is only one out of many causes of inconceivableness.

(2.) Whether a good proof or a bad one, inconceivableness of the contradictory is the best proof we can have, since all beliefs are in the last resort founded upon it.

Mill replies.—That this is not true is proved decisively by the facts that some Propositions are believed which are actually inconceivable. We cannot conceive that a body can act where it is not, that anything can be created out of nothing, that external thing can be bundles of sensations, and not objects are mere bundles of sensations, and not realities, external to us—yet all of these Propositions are or have been believed.

CHAPTER VI.

The Foundations of the Science of Number.

In the previous chapter we have seen that the primary truths of Geometry are axioms, and the hypothetical postulates contained by implication in the definitions. We have seen, moreover, that the axioms of Mathematics (as of other branches of knowledge) are arrived at and proved by experience, just as Propositions asserting any other law of Nature. We next proceed to discuss the *Science of Number*, which includes every branch of Mathematics not included in the *Science of Space or Extension* (that is, Geometry). Practically it consists of Arithmetic and Algebra.

I. *The fundamental Propositions on which the Science of Number is based, are:—*

1. *Definitions*, as { Two is one and one. Three is one and two.

2. *Axioms*, . . { The sums of equals are equal. The differences of equals are equal.

The definitions, like those of Geometry, define a name ("three" means "one and two"), and postulate or assume a matter of fact,—that collections of objects exist which impress the senses thus ∴, or thus . . ., and which may be resolved into two others respectively impressing the senses thus ., and thus . .

II. *These fundamental Propositions* (i.e., *the axioms and the postulates in the Definitions*) *of the Science of Number are Inductions,—generalisations from experience.*

Nothing need be added to the arguments already given under axioms of Geometry; and we proceed to discuss the

III. *Doctrine that the Definitions and theorems of the Science of Number are mere verbalisms.*

Putting out of view the two axioms, the advocates of this doctrine assert that the Propositions of this Science are simple transformations in language—substitutions of one set of words for another. That "three is two and one," say they, is not a statement of any external fact, but simply a way of saying that mankind have agreed to use the word "three" as exactly equivalent to "one and two," to call by the former name whatever is called by the latter more clumsy and less concise name; and so of every other numerical Proposition.

This view is supported by two arguments:—

First—We do not carry ideas of any particular things along with us when we manipulate algebraical or arithmetical symbols (as x or a).

Mill replies to this:—

(1.) That an examination of the mental phenomena involved in numerical processes shows that we have

been really dealing with *things* throughout; the *Symbols are things*, and our operations upon them express facts. For—(*a.*) These symbols will serve the purpose of things; and (*b.*) In the processes they are treated as things,—*i.e.*, the Propositions we make use of therein assert properties of *things*, and not of signs merely.

(2.) An examination of *the results* of numerical processes will often show us that we have really been dealing with *things*—for the facts at which we arrive in the conclusion are often by no means the same as the fact or facts from which we started.

Second—The Propositions of Number, when considered as Propositions relating to things, all seem to be identical Propositions.

Thus, "two and one are three," if applied to objects, seems to assert not mere equality, but absolute identity between the two collections of objects.

Mill replies—

It is true that the subject and predicate of a numerical Proposition may have the same *denotation* (*i.e.*, may denote precisely the same objects), but they have a different *connotation* (that is, they imply two different states of those same objects). The Proposition given asserts that a collection impressing the senses thus . ., and another thus ., if put together will impress senses thus .:.—these several impressions on the senses are what the names "two," "one," and "three" respectively connote; and the Proposition asserting that if the first two collections are put together they will impress the senses as in the third, though exceedingly simple and obvious, is yet not identical.

IV. *Under what circumstances the postulates in the Definitions of Science of Number are hypothetical.*

1. The Propositions of *pure number* (number merely as number) are true absolutely without any mixture of supposition. Number 3 always = number 1 + number 2.
2. But when from equality or inequality of number, equality or inequality in any other respect (as weight, size, &c.) is inferred, then the supposition or hypothesis that "all the numbers are numbers of the same or equal units" becomes necessary. We cannot be assured that 1 pound + 2 pounds = 3 pounds, unless we suppose 1 pound always to be the same.

V. "*The characteristic property, then, of Demonstrative Science is that it is hypothetical.*"

By this Mill means that—Demonstrative Science starts from the granting of certain fundamental suppositions, and then proceeds to trace the consequences of such assumptions, *i.e.*, what inferences may be drawn from them; leaving for subsequent separate consideration how far they are true, and what corrections must be made if they are not exactly true.

The inquiry, then, as to the inferences which can be drawn from assumptions or fundamental Propositions taken as settled, is what properly constitutes *Demonstrative Science.*

VI. *The "Reductio ad absurdum"* consists in thus assuming a Proposition which we wish to prove untrue, and then by inferring from it, and deducing an "absurd" consequence, showing its falsity.

An "absurd" Proposition nere means the contradictory of some Proposition which, by the conditions of the particular inquiry, is not to be questioned.

VII. *Some have said—That the ultimate proof of the validity of the Syllogistic process is dependent on a "reductio ad absurdum."*

That is, if any one admits the premisses of a Syllogism, yet denies the conclusion, by a "*reductio ad absurdum*" we can compel him to admit two contradictory Propositions—that is, one of the premisses and its contradictory. If he deny the Syllogism, he can be forced to a contradiction in terms.

Mill remarks—

This is not so, for since the validity of the Syllogism is denied, it is useless to attempt to prove it by a process which involves another Syllogism. The denier of the Syllogism can only be forced to an infringement of the fundamental axiom of ratiocination—"Whatever is a mark," &c.

VIII. *That Proposition, then, is logically*

"*necessary,*" to refuse our assent to which would be to violate the above axiom.

Nothing, therefore, is logically necessary but the connexion between conclusion and premisses.

Demonstrative Evidence is that from which anything follows by logical necessity, *i.e.*, as conclusion from premisses.

BOOK III.

INDUCTION.

CHAPTER I.

Preliminary Observations.

A *general Proposition* is one in which a Predicate is affirmed or denied of an actually or potentially indefinite number of individuals, viz., all existing or capable of existing in present, past, or future, which possess the attributes connoted by the subject-name. Or, it is

One which asserts that one phenomenon always accompanies (*i.e.*, is a mark of) another phenomenon.

[We must not, therefore, be misled by the mere verbal form of a Proposition. Thus, "All continents possess large rivers" is not a true logical general

Proposition, but only a bundle of four singular Propositions, viz., Europe possesses large rivers, Asia, &c., Africa, &c., America, &c. We cannot properly say attributes connoted by "continent" are marks of attribute "possessing large rivers;" the two only *happen* to be associated in the only cases of which we have knowledge, but if a new continent were raised, say from the bottom of Pacific Ocean, we have no assurance that it would contain large rivers.

On the other hand, "God is a being superior to man," is a general Proposition, as much to a Christian as to a polytheist, since it means—*whenever and wherever* we meet the attributes connoted by "God," there we shall meet attribute "superiority to man."

The distinction between a Proposition really general and one only general in its form, will easily be made if it be remembered that a true general Proposition asserts that one phenomenon is a mark of another phenomenon—and thus such a Proposition gives us a power of *predicting* that when or where we meet the former phenomenon we shall also meet with the latter. Thus, though we happen to be correct in saying, "All the apostles were Jews," we cannot predict from this that if an apostle were at any future time appointed, he would be a Jew,—the attributes connoted by "apostle" are not marks of the attributes connoted by "Jew."]

It may be well to draw attention to a slight ambiguity in the word "Induction," as used by Mill. Properly it means the inductive process or operation itself; but sometimes the *result* of that process,—the general proposition which is the conclusion of the operation. The context will show readily in which sense the word is used.

Induction is—(*a.*) an Inference, (*b.*) establishing a general Proposition, (*c.*) on the evidence of particular instances.

Notice the three clauses, (*a.*), (*b.*), (*c.*), since they all are necessary to constitute a true Induction. It must be an Inference, that is, the conclusion must be wider than the total of the premisses; and we say "establishing a general Proposition," because although we *may* argue from particulars to particulars, yet when we are logically warranted in doing this, we may also draw the general Proposition, as already explained. This is the sense, then, in which the expression must be understood: that in an induction we always *may* draw a general conclusion, though, as a matter of fact, we may content ourselves with simply inferring to a new particular case.

[*Another Definition of Induction*:—

"Induction is that operation of the mind by which we infer that what is true in a particular case or cases, will be found true in all cases which resemble the former in certain assignable respects."

This is essentially the same, but it is inserted for comparison.]

Importance of Inductive Logic; Induction the great subject of Logic.

All Inference (and, therefore, all proof, and all discovery of truths not self-evident) consists in Inductions, and in the interpretation and application of Inductions. All our knowledge, therefore, not intuitive, comes to us fundamentally from the same source.

The Logic of the Sciences is, therefore, the Logic of every-day life.

The same principles and processes apply to the inferences we are continually making in common affairs, and to the establishment of Scientific Principles.

CHAPTER II.

INDUCTIONS IMPROPERLY SO CALLED.

I. *Inductions improperly so called are*:—

1. *Mere Verbal Transformations.*
2. *Mathematical "Inductions"* } Nearly similar to 1.
3. *Inferences by Parity of Reasoning*, as— { (a.) Proving geometrical theorem by a diagram. (b.) Filling up terms of a series.
4. *Colligation of Facts.*

1. *Mere Verbal Transformations.*

That is, when we affirm of a class simply what has already been laid down as true of each and every individual in it separately; so that the conclusion

is only a summing up and reassertion (*i.e.*, "verbal transformation") of the premisses. Thus, Europe has large rivers, so Asia, so Africa, so America; ∴ All continents have large rivers. Paul was a Jew, Peter was a Jew (and so on through the whole list); ∴ All the apostles were Jews.

There is here no Induction, for—(1.) There is no *inference;* nothing more is stated in the conclusion than in the premisses; and (2.) The seemingly general Proposition is only a number of single Propositions written in a compendious and abridged form.

It may be added, this species of false Induction is the *only* form of Induction recognised by ordinary Logicians, who term it "True Logical Induction."

2. *Mathematical "Inductions."*

This process is of this kind: After having proved separately the following Propositions—

(1.) A straight line cannot cut a circle in more than two points.

(2.) A straight line cannot cut an ellipse in more than two points.

Similarly of the parabola and hyperbola—

Conclude that

"A straight line cannot cut a conic section in more than two points."

The conclusion in such cases is a really general Proposition, but the process is not Induction, for there is no *inference;* nothing but a summing up of the premisses, as in the last case.

3. *Inference by Parity of Reasoning.*

That is, when, though the conclusion arrived at is really general, yet we do not believe it on the evidence of

the particular case or cases themselves, but because we see that the same evidence which established the particular cases, will also prove every other case coming under our conclusion. Amongst examples of this are:—

(*a.*) *Proving a geometrical theorem by means of a diagram.*

Thus having proved that the three angles of the triangle A B C are equal to two right angles, we conclude that this is true of every triangle, not because we find it true of A B C, but for the same reasons which proved it true of A B C.

(*b.*) *Filling up the terms of a series when the law of the series has been ascertained.* (Prof. Clifford's exception to Induction — Binomial Theorem

4. *Colligation of phenomena or descriptions.*

Colligation of phenomena is the *forming a general notion or conception of those phenomena,* such general notion being constituted of the common attributes or properties of the phenomena colligated. This notion (the sum of the agreements of the phenomena) being expressed in words, constitutes a *description* of them. Thus, suppose we contemplate the whole animal creation, and discovering the points of agreement in, or the attributes common to every member thereof, we may by combining those points of agreement frame a general notion "animal," which notion unfolded in words would give a description of animal,—a notion and a description which would apply to every member of that department of creation.

Mill defines Colligation thus:—

Colligation is that mental operation which enables us to bring a number of actually observed phenomena

under a general description; or which enables us to sum up a number of details in a single Proposition.

There are two questions at issue between Mill and Whewell with respect to Colligation:—

1. As to the exact nature of Colligation, or the process of forming general notions.
2. The relation between Induction and Colligation.

The first is discussed in Book iv., chapter 2; the second we now proceed to examine.

II. *Relation between Induction and Colligation.*

According to Whewell—"Induction *is* the Colligation of phenomena by means of appropriate conceptions,—in short, Colligation is Induction."

Mill replies.—The two processes are quite distinct; Colligation, or the formation of a general conception of the phenomena to be investigated, is *a necessary preliminary to Induction;* but Induction is something more than Colligation; for—

1. There is *no real inference* in mere Colligation.
2. Successive Colligations, though conflicting, may yet all be correct as far as they go; that is, they may all correctly represent the *facts observed* at the time they were respectively *framed.* Now, it would be absurd to assert that conflicting Inductions could all be true. Thus, the successive

description, if I succeed in conveying to him the same notion as he would have had of the facts if he himself had observed them in my stead. But if new facts are discovered, a new, and perhaps different, notion or description will be required, which again may have to give place to a third, differing from both, if new facts in due course come to light; yet every one of these notions and descriptions will have been in its turn correct *as a description* of the facts known at that time.

3. Colligation only *describes;* Induction, besides incidentally describing, also *predicts and explains.* Note

Whewell, in fact, confounds *conception,*—the process of forming general notions (or, as he calls it, "*Colligation*") with *Induction.* When we conclude by an Induction that "All men are mortal," he would represent the process as consisting simply and solely in framing a general notion of man, which general notion should include "mortality," and then comparing this general notion with observed facts to see if it agrees therewith; and if not, framing another and different notion, comparing it in like manner; and so on till we find a notion which *will* correspond to the facts. It may at first sight seem strange that two such apparently distinct processes as Induction and Conception should be thus confounded; but compare Bain's "Senses and Intellect" under "Similarity" (Reasoning and Science).

CHAPTER III.

GROUND OF ALL INDUCTION.

I. *Fundamental axiom or ground of Induction.*

There is an assumption implied in every case of Induction, and which assumption is found to be true as a matter of fact:—

Loosely expressed—This fundamental assumption is, that *the course of Nature is uniform*—as far, at least, as regards the phenomena we are concerned with in the particular Induction.

More accurately—That what is true in certain cases is true of every other case resembling the former in certain assignable respects.

II. *How this assumption is involved in any Induction.*

An Induction may be thrown into the form of a Syllogism by supplying a major premiss, thus:—

Whatever is true of A, B, C, is true of all men,
A, B, C are mortal,
∴ All men are mortal.

Now, it is evident that this major premiss is nothing else but an assertion of the uniformity of nature in so far as regards the phenomena we are concerned with. "Uniformity in nature" means that what we find in one or more cases we shall continue to find in similar cases, which is exactly our major premiss.

But this major premiss resembles the major premisses of any other Syllogism; that is, it is no part of the evidence which proves the conclusion, but only a mark that there is sufficient evidence to prove the conclusion, so that if false the conclusion is fallacious. In fact, as already explained, in connexion with the theory of the Syllogism, both the conclusion and the major premiss are alike conclusions from the antecedent observed particular cases. We see A, B, C, and everybody else in whose case the experiment has been fairly tried, die,—this is our evidence, and from it we conclude "All men are mortal;" but it is that very same evidence that gives us our assurance that "what, in this respect, is true of A, B, C, is true of all men," in other words, our observed cases prove to us that nature is uniform in respect of the connexion between humanity and mortality. (*See* p. 67.)

III. *How uniformity of Nature is proved.*

This ultimate principle is a generalisation from all our Inductions, it is a conclusion by an Induction, "*per enumerationem simplicem*," from a large number of Inductions.

For example, suppose we knew nothing of the principle, *i.e.*, did not know whether Nature was uniform or not, yet in the Induction in II., we could conclude that Nature was uniform in respect, at least, of the connexion between men and mortality; so also in a second Induction we might prove Nature uniform in another respect; in a third, and so on.

As fresh instances of proved uniformity were added to our list, we might begin to suspect that Nature was always uniform; new cases of Induction constantly being made, each in its own sphere proving Nature uniform, would proportionally strengthen that suspicion; and when finally age after age passes away, and Inductions innumerable are made, *every one* of which adds its item of proof without a single contradictory instance (that is, an instance of Nature capriciously varying being found), the inference of the universality of the principle is irresistible. Before proceeding farther it is necessary to explain:—

Induction "per Enumerationem Simplicem," is thus defined by Bacon.

"*Ubi non reperitur instantia contradictoria,*" *i.e.*, Induction, because we have never found an instance to the contrary. It is an argument from simple, unanalysed experience; its formula being, "Such and such has always been found to be true, no instance to the contrary has ever been met with; therefore such and such is true." All crows hitherto observed have been black, no crow of any other colour has ever been seen, therefore all crows are black.

Mill's definition is, the ascribing the character of a general truth to any proposition which happens to be true in every instance we have known of,—*i.e.*, to which we do not happen to know any exception.

It is the sort of Induction natural to untutored minds, and is usually, but not always very fallacious. The following is the

Condition which renders an Induction per Enumerationem Simplicem a valid process.—(*See* pp. 177 and 181.)

We must know that if any exception ever had occurred we should be aware of it; in other words, precisely in proportion as its subject matter is limited and special, so is the process unreliable.

This necessary assurance we cannot in the great majority of instances obtain; yet it is the fact that there do exist certain remarkable cases where, having this certainty, an Induction by simple enumeration amounts to a rigorous proof, indeed the only proof of which these cases are susceptible. These are (1.) Fundamental principles of Mathematics; and (2.) The principle of the Uniformity of Nature.

The axiom of the uniformity of Nature, then, is proved by this form of Induction; the evidence consisting in this, that the principle has been found true in every legitimate Induction hitherto made, and never once false; while, at the same time, from the fact that these innumerable Inductions cover the whole field of Nature's operations, we are entitled to conclude that any real exception must have come under our notice.

IV. *The chief merit of Bacon as regards Inductive Philosophy* lay in his pointing out the insufficiency of this loose and merely passive mode of Induction, and the essential importance of an active interrogation of Nature by experiment.

CHAPTER IV.

Laws of Nature.

I. *The general regularity in Nature is an aggregate of particular uniformities called Laws.*

("*Uniformity.*"—A uniform conjunction of phenomena either by way of co-existence, the two phenomena always being found together, or by way of sequence, one phenomenon always being followed by the other.)

That the general uniformity of Nature is made up of uniformities in particular respects requires no illustration, being self-evident.

II. *Laws of Nature.*

(1.) *In the loose sense.*—A law of Nature is a proposition expressive of any sufficiently well ascertained uniformity.

(2.) *In a stricter sense.*—A law of Nature is any established uniformity which cannot be accounted for by, or resolved into, simpler uniformities. The expression "Law of Nature" is commonly employed with a tacit reference to the true signification of "law," the expression of the will of a superior; and hence is employed to designate such uniformities only as might be regarded as separate expressions of the creative will; and is not usually applied to such uniformities as can be shown to be mere

results of such fundamental laws. Thus, that "the mercury will rise in an upright exhausted tube, whose orifice is immersed a little beneath its surface," is a proved uniformity, which, however, may be resolved into two more fundamental principles—gravity, and the uniform transmission of pressure by a liquid; and as these two cannot be resolved into more fundamental principles, they rank at present as laws of Nature, while the derivative law or uniformity does not.

III. *Scientific Induction must be grounded on previous spontaneous Inductions.*

Spontaneous Inductions are those which are so palpable as to be made without conscious effort,—inductive inferences which force themselves upon men's minds,—as that "fire burns," "water quenches thirst," and so on. Now such Inductions as these give us that insight into the order of Nature which is necessary before we can lay down the principles of Induction; if a rational being were suddenly created and dropped upon our earth, however great his intellectual powers, it would be utterly impossible for him to frame inductive canons; he would not know whether caprice or uniformity prevailed, and if uniformity, under what circumstances it might be expected to be manifested. There is nothing impossible in the supposition that the arrangement might be such that what has the properties of iron at one moment might have those of ice the next; that, in a word, uniformity might be replaced by the wildest caprice, or that the uniformity, even if found, might differ widely from that which we actually experience. *In fact, we require experience* Zote

to show us how far and under what conditions experience is to be relied upon.

IV. *The stronger* (i.e., *more certain*) *Inductions are the tests to which we endeavour to bring the weaker.*

Suppose, for instance, that we possess a strong Induction to this effect,—"every effect must have a cause," it is evident that if by any means we can bring a weaker generalisation within this better established law,—*i.e.*, if we can show that either the weaker generalisation must be true or our strong Induction must be false,—the weaker is at once raised to the same degree of certainty with the stronger.

Since, then, the logical method of proving a generalisation is thus to bring it within a more certain generalisation, the inquiry necessarily arises—

Do any great Inductions exist, thus fitted to be ultimate tests of all others?

There are such Inductions, certain and universal, and it is because there are such that a Logic of Induction is possible.

The universal *Law of Causation* is such an Induction, and the four Inductive methods of Mill are simply expedients for bringing weaker generalisa-

CHAPTER V.

Law of Universal Causation.

I. *Law of Universal Causation.*

Every phenomenon which has a beginning must have a cause; and it will invariably arise whenever that certain combination of positive facts which constitutes the cause exists, provided certain other positive facts do not exist also.

This law contains two clauses:—

(1.) That every phenomenon which has a beginning must have *some* cause.

(2.) Given the cause, the effect will invariably follow, provided that counteracting causes do not exist.

II. *Definition of Cause.*

(1.) If we regard cause as including *all* the antecedents, both positive and negative ("positive"=what must be present; "negative"=what must be absent).

A cause is that assemblage of phenomena, which occurring, some other event follows, invariably and unconditionally.

This is what some express by saying that an effect follows its cause "necessarily." To be a cause, it is not enough that the sequence is *invariable;* night is an invariable sequence of day, but day is not the cause of night. The sequence must be *unconditional* also,—given day, night should follow, whatever we choose to suppose about other things (as, for instance, that rotation of earth should cease), to justify us in calling day the cause of night.

The *negative conditions* of an effect may be summed up in this,—absence of counteracting or preventing causes.

(*a.*) *Counteracting causes.*—Most causes counteract the effects of other causes by the operation of the very same law as that by which they produce their own effects. Each law is fulfilled,—each cause in reality has its effect, but the effects neutralise each other more or less. Thus, if a spout supply a cistern at a certain rate, while a precisely similar spout empties it simultaneously, the effect, the filling of the cistern, is defeated, though the cause, the influx of water, cannot be spoken of otherwise than as really producing its own proper effect.

(*b.*) *Preventing causes.*—Some causes seem, however, to be simply preventive,—*i.e.*, destroying an effect, (not by producing their own, but) by simply arresting it. *Opacity* is a phenomenon of this kind.

III. *Popular distinction between the "conditions" and the "cause" of an effect unphilosophical.*

Strictly speaking, the cause is the sum total of all the conditions or circumstances necessary for the pro-

duction of the effect—the aggregate of the antecedents thereof. In popular language, however, it is usual to single out one of these antecedents *as the cause*, the remainder being termed *conditions* of the effect. Thus, suppose a stone to be dropped into water and allowed to sink,—the sinking of the stone is an effect, the antecedents being (1) the mutual attraction between stone and earth; (2) the stone being within the range of that attraction; (3) the specific gravity of the stone being greater than that of the water; and, finally, the negative condition, absence of support for the stone. Now, in common discourse, any *one* of these conditions,—the entire sum or aggregate of which is properly the cause,—may be called *the* cause of the sinking of the stone. Even a mere negative condition (*i.e.*, the *absence* of something) is often spoken of as if the positive cause of an effect (as, the absence of the sentinel was the cause of the army being surprised), whereas it is evident that no mere negation can produce an effect, but can only not hinder its being produced.

The distinction thus popularly made is usually based upon either :—

(1.) That one of the antecedents which comes last, and is thus *an event* completing the sum of conditions which forms the cause, and upon which the effect immediately follows, is termed *the* cause.

(2.) That one of the antecedents which is most peculiar and special to the aggregate of antecedents, is often popularly *the* cause.

(3.) So also that one of the antecedents which is least likely to be known to the hearer.

IV. *The Universal Law of successive phenomena is the Law of Causation.*

(This does not mean that two successive phenomena are necessarily cause and effect,—day and night, for instance, are not; but any phenomenon which succeeds another must be a phenomenon having a beginning, and therefore a cause, and consequently must come under the law of Causation.)

Phenomena in nature may stand to each other in two relations,—that of *Simultaneousness* (*Co-existence in Time*), or that of *Succession*. The most valuable truths with which we have to do are truths of Succession: on our knowledge of *these* depends all our power of foreseeing future facts, and of influencing these facts for our own benefit. Hence we see why the *main* business of Inductive Logic is with cases of Causation—with determining what are the effects of given causes, and what the causes of given effects.

[V. *Distinction between "agent" and "patient" merely verbal.*

Many make a distinction between the thing acting (*the agent*) and the thing acted upon (*the patient*). Thus, if a man be poisoned by prussic acid, the poison would be reckoned as an "agent," the nervous system of the individual as the "patient."

This distinction is, however, merely in language; "patients" are always "agents" which have been implied in the words describing the effect. It is evident, for instance, that the peculiar properties of the nervous system are as much involved in, are as much conditions of, the death as the properties of prussic acid.]

VI. "*Permanent Causes*" *or* "*Primitive Natural Agents*" include all substances and phenomena which do not begin to exist,—*i.e.*, which might, for aught we can see in them to the contrary, have existed from all eternity, and may never have had a beginning at all.

Such may be either:—

(1.) *Objects*—the sixty-three or more elementary bodies, with their various properties and the combinations of such found in nature,—the atmosphere, water, &c.; the heavenly bodies, sun, moon, and stars.

(2.) *Events*—*i.e.*, cycles of events—periodical cycles being the only form in which an event can have permanence. Such are the rotatory and orbital motions of the earth, of the moon, &c.

Of these original Causes it may be observed:—

(1.) We cannot tell why any one of them exists at all; we can give no account of their origin.

(2.) We cannot tell why they exist in a particular manner—why one is found in one place, another in another.

(3.) No law of their mode of distribution can be discovered.

(4.) Every phenomenon which has a beginning, every *caused* phenomenon,—*i.e.*, every phenomenon except the primeval causes themselves,—must arise or have arisen immediately or remotely from one of these primeval causes, or from some combination thereof.

CHAPTER VI.

ON THE CONJUNCTION OF CAUSES.

I. When two or more Causes act together so as to intermix or combine their effects, one of two things may happen, either :—

1. The joint effect is of the same kind with the separate effects; the laws work together without alteration. In this case we may speak of the mixed effect as *consisting of* the separate effects. This constitutes "*Composition of Causes*," and such an effect is termed a "*Compound Effect*."

   For example, suppose a force acting on a particle in the direction of the north, and another tending to pull it to the west, the two forces are two conjoined causes, and the effect, which is a force lying along diagonal of their parallelogram, is of the same kind with the separate effect of each cause. The effect of each of the two forces is, in fact, found in the conjoined effect of both. This is, therefore, a case of "Composition of Causes," and the effect is "Compound."

2. The joint effect is not of the same kind with the separate effects; the separate effects of the causes disappear, and a totally new set is developed by their combination. Here we may speak of the conjoint effect as being *generated or produced by* the simple effects. This is the case of "*Combination of Causes*," and such an effect is a "*Heteropathic Effect*."

   For example, the substance iodine manifests, or is a cause of, certain properties or effects—dark colour, peculiar smell, metallic taste, volatility. So also potassium causes us to feel a metallic lustre, &c. But if these two causes, iodine and potassium, be united, we find hardly any trace of the effects which they produced when separate; those effects indeed are gone, and are replaced after the union of the causes by a totally distinct set—white crystalline structure, solubility in water, &c. So, again, the properties of a mixture of oxygen and hydrogen are chiefly the sum of those of the separate gases conjoined—a *compound* effect; but let the mixture be exploded, the two causes, oxygen and hydrogen, combine, but no trace of the properties of the separate gases can be found in water. This effect, therefore, is *heteropathic*.

II. *Composition of Causes.*

1. The causes may act in the same or in different directions,—two forces may pull a body along the same, or along intermediate, or along opposite lines. Two spouts may either both fill, or one may empty and the other fill, a cistern. Causes may, in fact, appear to annihilate each other's effects,—apparently producing no effect at all,—as when two equal forces act in opposite directions upon a particle, yet each cause really exerts its full efficacy according to its own law.
2. The Composition of Causes is the more frequent case, not only absolutely in the case of simple causes, but for the reasons given below in III. and IV.

III. *The total effect of conjoined Causes may*

be partly compounded, partly heteropathic, and, in fact, is never wholly heteropathic.

There are no cases of causation, from conjoined causes, in which the resultant phenomena do not in some respects obey the principle of the Composition of Causes. The *weight* of a combination of iodine and potassium, for instance, is always the sum of the weights of the separate ingredients.

IV. *Heteropathic phenomena, when they act together, may compound their effects.*

Laws generated by combination may act with one another on the principle of composition. As a single example, iodide of potassium and of water—both generated heteropathically—may, if mixed, give the sum or aggregate of their separate effects, as regards taste, chemical reactions, &c.

V *That effects are proportional to their Causes*—

Is, when true, a case of the composition of causes, a cause then being compounded with itself. If, for example, a column of mercury be heated through 1°, a certain expansion follows, through 2° twice as much, and so on, the effect being so far proportional to the cause. It is clear, however, that in this case, the total effect of a rise through several degrees is compounded of the separate effects of a rise through each degree. If, however, the cause be augmented beyond a certain limit, the proportionality ceases, the mercury being converted into vapour,—a hetero-

pathic effect. So we shall always find that the axiom holds good as long as every fresh increment of the cause compounds its effect with that of the preceding parts of the cause, but that it fails the moment the composition of causes fails, and the effect becomes heteropathic.

CHAPTER VII.

Observation and Experiment.

I. The order of Nature, looked at as a whole, presents a vast mass of causes followed by a vast mass of effects, and, therefore, Inductive inquiry,—having for its object the ascertaining what causes are connected with what effects, and what effects with what ca[illegible] some sort a process of Analys[illegible] steps—(1.) The men[illegible] actual separation [illegible] [illegible]ces.

The whole order of nature, as perceived at the first glance, consists of a great mass of phenomena, followed in the next instant by another great mass of phenomena. The first step, then, consists in learning to see in the aggregate antecedent a number of separate antecedents, and in the aggregate consequent a number of separate consequents.

The second step is an actual separation of the elements of complex phenomena.

We must *obtain* some of the antecedents apart to be able to try what will follow from them, or some of the effects apart to find out by what they were preceded.

The third step, determining what antecedents and what consequents are connected,

Is accomplished by *varying the circumstances,*—that is, by obtaining instances of the phenomenon we are investigating, which, by differing in some of their circumstances, will throw light on the inquiry.

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upon what it is *in itself*, and not upon the mode in which it is obtained, yet there are important *practical* distinctions between observation and experiment, which it is necessary to notice.

III. *Experiment* is our resource when we wish to determine the effect of a given cause, for we can take a cause and try what it will produce, but we cannot take an effect and try what it was produced by.

Advantages of Experiment are:—

(1.) It enables us to *multiply our instances* indefinitely.

(2.) To *isolate* the phenomenon we are studying.

(3.) To *vary* the surrounding circumstances indefinitely, and thus, amongst other things,

(4.) To obtain *the precise sort* of instances we require.

(5.) To these may be added, that, since our only way to *prove* that one thing is the cause of another, is to take the supposed cause and try whether it will produce the effect, and since we cannot generally do this by observation, *experiment is usually our only means to prove causation.*

IV. *Observation* is chiefly applicable when...

causes, we cannot produce cholera artificially, and our only resource is to wait till nature produces instances for us, by observation of which we may hope to discover by what they have been invariably preceded.

Observation alone, without aid from experiment, can rarely prove cause and effect. It may show us that two phenomena are *invariably conjoined*, that where we find one we shall also find the other, but it will not go beyond this; we cannot be sure that the two phenomena are cause and effect: they may, for example, be both effects of some common cause. To *prove* cause and effect, we must take the cause and try what effects it produces,—a matter of experiment, as already said.

V. From this contrast between observation and experiment, an important conclusion follows:—

That in the sciences in which artificial experiment is impossible or very limited, direct Induction is practised at a disadvantage, amounting generally to impracticability.

The methods of such science must therefore be chiefly Deductive.

CHAPTERS VIII. AND IX.

THE INDUCTIVE METHODS.

[*Preliminary Remarks.*

1. Mill speaks of these as the "Four Methods of Experimental Inquiry," and these are:—

1. The Method of Agreement and the Joint Method.
2. Of Difference.
3. Of Residues.
4. Of Concomitant Variations.

"*The Joint Method*" is not reckoned separately, inasmuch as it is really an employment of the two forms of the method of agreement together,—the Positive and the Negative. It is termed by Mill either the "Joint Method of Agreement and Difference" (or shortly, "The Joint Method"), or "The Indirect Method of Difference."

2. When these are spoken of as Methods of *Experimental* Inquiry, the term experimental must be taken as equivalent to *experiential*,—Methods of Inference from experience generally, and not merely from experiment in the strict sense. The Method of Agreement, for example, usually derives its instances from observation.

3. The Method of Agreement and of Difference

are the two fundamental Methods; the others being only special forms of one of these, or of both together. Thus:—

1. Method of agreement, (where we find cause we find the effect, and *vice versâ*,)	Includes also the Joint Method (in part—the negative partaking of Method of Difference).
2. Method of Difference, (where one is absent or removed, the other is also.)	Includes also Method of Residues, Method of Concomitant Variations.

It is easy to see how the Method of Concomitant Variations partakes of the nature of the Method of Difference; two phenomena (A and a) are found conjoined, and the (partial) removal or (partial) adding on of one is followed by a corresponding change in the other, which is essentially the Method of Difference.

4. In the exposition of the Inductive Methods, Mill takes the simplest possible case,—that is, he supposes every effect.

1.) To always have exclusively one and the same cause, and (2.) To be always distinct from—not in any way intermixed with—any other coexistent effect. In what way it becomes necessary to modify our proceedings when we come to the practical use of the methods is discussed in Chapter X.]

I.—Method of Agreement.

Canon.—If two or more instances of the presence of the phenomenon in question have only, in common,

the presence of one other circumstance, that circumstance, in the presence of which alone all the instances agree, is the cause or effect of the given phenomenon.

Its principle is—that of comparing different cases in which the given phenomenon occurs, in order to discover in the presence of what these instances agree.

Exemplification.

Suppose the given phenomenon is *cholera*, and we wish to ascertain its cause: by this method we should have to compare a number of instances of cholera, to determine by what it had invariably been preceded. It is clear that the cause of cholera must be amongst these invariable antecedents; and *if* we can be sure that in each case we know everything which has preceded (or all the antecedents of) the attack, and *if*, in a number of cases, only *one* circumstance can be found which has invariably preceded, that one must be the cause of cholera.

Remarks:—

1. *The possibility of the plurality of causes* introduces the possibility that the two phenomena, thus apparently connected, may only be conjoined casually or by chance. This is called by Mill "*the characteristic imperfection*" of the Method of Agreement.

Most effects may be produced by a plurality of causes,—*i.e.*, by different causes in different cases; thus, the phenomenon "*death*" may be caused in one case by disease, in another by poison, and in a

third by injury. This possibility does not, however, radically vitiate the method, but only renders it necessary that its first results should be corrected by the process for the "*Elimination of Chance*" (which see).

2. There is, however, another imperfection in this method, which prevents us from ever proving by it more than that two phenomena are invariably conjoined; it cannot demonstrate that they are cause and effect. This, which may be termed "*the practical imperfection*" of the method, is *the impossibility of assuring ourselves that we know all the antecedents in our instances.*

To take our previous example, we might find that the drinking of a certain sort of impure water was alone an invariable antecedent, as far as we can see of cholera. Suppose, further, we had assured ourselves that these two phenomena were not merely accidentally found together in our instances, still we could not be assured that they were cause and effect. The impure water and the cholera attack might both be results of some obscure cause which had wholly escaped our observation—some unknown atmospheric state, for example, forming an antecedent in our instances, of which we were ignorant. *The possibility of the presence of unknown antecedents*, then, is the reason why the Method of Agreement can only yield empirical laws, and cannot prove causation, and is, therefore, chiefly useful as affording suggestions, or as an inferior resource where better methods are impracticable,—*i.e.*, where artificial experiments cannot be made.

3. Since this method is our chief resource when we wish to determine the causes of a given effect, it mostly makes use of instances obtained by observation.

4. This method does not require instances of a very special and definite kind; any instances whatever, in which the phenomenon occurs, may be examined for the purposes of this method.

Negative Method of Agreement.

The *Canon* is the same as that of the Positive form of the method, with the substitution of the word "absence" for "presence," wherever it occurs.

The *principle* is that of comparing different instances of the absence of a phenomenon, to discover, if possible, in the absence of what other thing alone those instances agree. In other words, to fix upon the one thing alone which is never found where our phenomenon is absent.

The negative Method of Agreement is not affected by the possibility of a plurality of causes, but it is affected by the possibility of the absence of unknown antecedents, just as the positive form is by the possibility of their presence.

The further consideration of this form of the Method of Agreement is deferred till we come to speak of the "Joint Method;" for though it is true abstractedly that it might be worked alone without the positive method, yet practically, it is quite impossible to make much use of it by itself. The difficulty of showing that the instances agree in the absence of one thing only, is almost insuperable. But when the positive method has suggested a cause, we can then inquire, with some chance of success, whether that is not the only thing universally absent when the effect is absent.

II.—Method of Difference.

Canon.—If an instance in which the phenomenon in question occurs, and an instance in which it does not occur, have every other circumstance in common save one, that one occurring only in the former,—that circumstance in which alone the two instances differ, is the effect or the cause, or a necessary part of the cause, of the phenomenon.

The *principle* is that of comparing an instance of the occurrence of a phenomenon with a similar instance in which it does not occur, to discover in what they differ.

Remarks:—

1. This method is more particularly a method of artificial experiment (its ordinary use being to compare the condition of things before, with those after, an experiment), because—
2. It is commonly employed to determine the effects of given causes; and because—
3. The instances which it requires are rigid and definite—they must be exactly alike, except that in one the phenomenon must be present and in the other absent.
4. If this method is inapplicable, it is usually because artificial experiment is impracticable.
5. It is the only method, of direct experience, by which laws of causation can be proved.
6. If the instances fulfil exactly the requirements of the Canon, this method is perfectly rigorous in its proof.

7. Many of our inferences in daily life are simple applications of the Method of Difference.

   Thus, a man in full life receives a shot in his heart and becomes a man dead. We infer that the wound caused death, because it is the only circumstance in which the case in which death is found differs from the case in which death is not found.

III.—The Joint Method.

Canon.—If two or more instances of the presence of the phenomenon in question have only in common the presence of one other circumstance; while two or more instances of the absence of the phenomenon have in common the absence of that circumstance only,—that circumstance in which alone the two sets of instances differ, is the effect or the cause, or a necessary part of the cause of the phenomenon.

Remarks:—

1. The Joint Method is really a double employment of the Method of Agreement, thus:

   We observe a number of instances in which the phenomenon is present, and find them to agree only in the presence of a given circumstance. (Positive form.)

   We observe a number of instances in which the phenomenon is absent, and we find that that same circumstance is the only thing which is uniformly absent. (Negative form.)
2. It is called, also, "*The Indirect Method of Difference*," because the negative instance is got not by direct

experiment, but *indirectly*, by showing what *would* be the result if experiment could be made.

3. The Method of Difference compares two instances; the Joint Method compares two *sets* of instances. The proof derived from one set is independent of that derived from the other, and corroborative of it. Still both together do not amount to a proof by the direct Method of Difference, on account of the possibility of the presence or of the absence of unknown antecedents in the positive and negative sets respectively.

4. The Joint Method is, however, a great extension of the Simple or Positive Method of Agreement; having this great advantage over it, of not being affected by the possibility of the plurality of causes.

IV.—Method of Residues.

Canon.—Subtract from any phenomenon such part as is already known to be the effect of certain antecedents, then the residue of the phenomenon is the effect of the remaining antecedents.

Exemplification.—Suppose we have several phenomena A B C followed by several others a b c, and that we know A to be the cause of a, and B of b, then C must be the cause of c.

Remarks:—

1. This method is, in fact, a modification of the Method of Difference; but the negative instance (*i.e.*, where phenomenon is absent) is obtained by Deduction, not by direct experience. The Deduction being

this,—from the known effects of A and B separately, we *infer* their effect conjointly, and subtract this effect from the total effect, a b c.

2. This method would be equally rigorous with Method of Difference, if (1.) we could be certain of the total effects of the known antecedents (A and B), and (2.) that the remaining antecedent (C) is the only one present.

3. This being generally impracticable, we must complete the evidence derived from this method, either—

   (1.) By applying Method of Difference,—*i.e.*, obtaining supposed cause (C) separately, and trying its effect; or

   (2.) By the Deductive Method,—*i.e.*, we must account for C's agency when suggested, and infer it deductively from established laws.

4. This method is the most fertile in unexpected results.

V.—Method of Concomitant Variations.

Canon.—Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it by some link of causation.

The principle is—that even if we cannot remove an antecedent *altogether*, yet we may be able to modify it in some way short of its total removal.

Its Axiom is—Anything upon whose change, the change of an effect is invariably consequent, must be the cause or be connected with the cause of that effect.

Remarks :—

1. The changes or variations with which this method is chiefly concerned are either—(1.) Changes in quantity, or (2.) Changes of position in space.
2. To logically infer causation through this method, we must first determine that *the variations in the two phenomena are really concomitant.* This is proved by the Method of Difference,—that is, we retain all the other antecedents unchanged, while the particular one is subjected to the requisite variations.
3. This method may *usefully* follow the Method of Difference, to determine according to what law the quantities or relations of the effect follow those of the cause.
4. The *most striking application* of this method is to cases where we have to determine the effects of those of the Permanent Causes, which we cannot wholly remove.

   Such causes are—*the earth*, with its gravitative, magnetic, and other properties; *the sun, moon*, and *stars*, with any known or unknown properties they possess. These causes we can never *wholly* remove, but we can *modify* them,—we can get nearer to or farther from the centre of the earth, we can remove a magnetic needle from one place to another, and by such methods, varying the supposed cause, and noticing the consequent variations of the effect, we are able to determine the effects of these irremovable permanent causes, and separate them from the effects of other causal agencies.
5. The *most satisfactory application* of this method is in cases where the variations in the cause are variations in *its quantity.*

   This kind of variation in a cause is generally accompanied not only by variation in the effect, but by composition of causes, with a *proportional* variation thereof.

We may have *two cases* of this relation of variation :—

(1.) Where cause and effect vanish together.
(2.) Where they do not vanish together,—*i.e.*, where one is reduced to 0, the other has still some positive value.

Two precautions are necessary in drawing conclusions from this kind of concomitant variation :—

(1.) We ought to be able to determine the absolute quantities of the antecedent (A) and consequent (a).

For if we do not know the absolute quantities of A and a, we cannot tell the exact numerical relation according to which these quantities vary. We cannot say that we have twice, three times, &c., as much of a thing, unless we know the quantity of once the thing.

(2.) We must remember that the law which the quantities seem to follow within the limits of our observation may not hold beyond those limits.

VI.—General Remarks.

1. These four Inductive Methods are the only possible modes of inquiry by experience, or *à posteriori.* These, therefore, with such assistance as can be obtained from Deduction, constitute the available resources of the human mind for determining the Laws of the Succession of Phenomena.

2. *Whewell makes the following objections to these Methods:—*

(1.) *They assume the very thing which is most difficult to obtain,—the reduction of an argument to a formula.*

Mill replies:—

This objection is exactly analogous to that brought against the Syllogism, by those who said that the great difficulty is to get your Syllogism, not to judge of it when obtained. As a matter of fact, both of these objections are true, but still the canons and formulæ fulfil their *logical* function, that is, they enable us to judge of evidence when found,—the very office of the Science of Logic, and by no means a superfluous one, as the commonness of false inferences testifies.

(2.) *No discoveries have ever been made by their means.*

Mill replies:—

This objection proves too much, for since the methods are formulæ of the only possible modes of inference from experience, this assertion is equivalent to saying that no discoveries were ever made by experience.

CHAPTER X.

PART I.—PLURALITY OF CAUSES.

I. By *Plurality of Causes* is meant simply this,—that a given effect may arise from different causes in different cases; thus, the phenomenon "*death*" may be caused in one case by disease, in another by violence, in a third or fourth by poison or old age.

II. *The possibility that the effect we are investigating may have a plurality of Causes,* leads to "*the characteristic imperfection*" of the Method of Agreement (in its positive form at least). p 117

Thus, suppose we have a group of causes, $A\ B\ C$, followed by a group of effects, $a\ b\ c$, and $A\ D\ E$ by $a\ d\ e$ (the fact being that A is the cause of a, B of b, and so on, though we are not supposed to know this); by the Method of Agreement we conclude that A is the cause of a,—since B or C cannot be, because absent in second instance, nor D nor E because absent in first; but the moment we admit that a might have a plurality of causes, this conclusion fails,—it might be, then, produced by B or C in the first, and by D or E in the second case.

III. *How to correct the uncertainty arising from this cause.*

The method of Agreement is not *radically* vitiated by this imperfection; for the two phenomena found together must either (1.) have no connexion,—*i.e.*, be conjoined by chance, or (2.) must have some connexion.

(1.) By merely multiplying instances of the same kind, we shall get data for determining *whether the coincidences are more frequent than chance will account for;* if so, we may conclude there is some connexion (between our A and a).

(2.) If, however, we sufficiently multiply and *vary* our instances (*i.e.*, get them as the canon prescribes, agreeing only in one other circumstance (A) besides a, the phenomenon in question), having $A\ B\ C$ followed by $a\ b\ c$, $A\ D\ E$ by $a\ d\ e$, $A\ F\ G$ by $a\ f\ g$, &c., we may be sure either :—

(*a*.) That a has as many causes as there are instances,—that is, that A and a are only conjoined by chance. Remedied as in (1).

(*b*.) That A and a are joint effects of some *unknown* antecedent existing in all our instances; or

(*c*.) That A is the cause of a.

We can never wholly get rid of the possibility of the presence of an unknown antecedent, and all therefore we can conclude by this method is that A and a are found together,—where A is we may expect a, or *vice versâ*.

[IV. *We may recapitulate here the use of mere number of instances of the same kind :—*

1. They enable us to have proportionate assurance that no error has been committed in the observation of the particular facts.
2. They furnish data for eliminating chance,—*i.e.*, for showing that the conjunctions between two phenomena are more frequent than mere chance will account for.

When these two objects have been fully attained, nothing whatever is added to the certainty of our conclusion by mere repetition of similar instances.]

V. *To determine Causes of a given effect, producible by a plurality of Causes.* This is done either :—

(1.) By separate inductive inquiries, each cause being tested by a separate series of investigations; or,

(2.) By collecting a number of instances of the occurrence of the effect, and finding that while they agree in no one antecedent, yet they always agree in the presence of one out of a certain number thereof. Thus, the effect "death" is always preceded by disease, old age, poison, or violence.

VI. *Plurality of Causes does not affect the Negative Method of Agreement nor the Method of Difference.*

1. In *negative method* we show that instances in which a is absent agree only in not containing A. Now, if

this be so, A must not only be the cause of a, but the only possible cause. For if a is absent, all its causes must be absent in every case, but the *only* thing uniformly absent in our instances is A, $\therefore A$ is the only possible cause of a.

2. The Method of Difference is obviously not affected. If we have two cases,—$A\ B\ C$ followed by $a\ b\ c$, and $B\ C$ by $b\ c$ (*i.e.*, A being taken from $A\ B\ C$ gives $B\ C$ followed by $b\ c$, or being added to $B\ C$ followed by $b\ c$, gives $A\ B\ C$ by $a\ b\ c$), it is certain A is the cause of a in *that* case, whatever other causes of a may exist in other cases.

PART II.—INTERMIXTURE OF EFFECTS.

I. We have already explained what is meant by *a complex or intermixed effect* (see Chap. vi. Book iii.).

It is an effect resulting from the conjunction of several causes. Thus, if a person suffering from severe illness were to take some poison, his death might be a complex effect, resulting partly from disease, partly from poison; so in the parallelogram of forces or velocities, the movement of the particle along the diagonal is a complex effect—the intermixture of the effects of two causes, viz., the two forces tending to carry it along the two sides of the parallelogram.

We have seen also that such complex effects are divisible into two distinct classes—(1.) *Compound Effects*, where the separate effect of each of the causes really continues to be produced, and these separate effects

unite into one aggregate or sum—the complex effect and (2.) *Heteropathic Effects*, where the separate effect of each cause ceases entirely,—a perfectly different phenomenon resulting from the conjunction of the causes.

There is, however, a special form of Heteropathic Effects, which require to be separately noticed—*Transformations, where cause and effect are mutually convertible*—*i.e.*, where we can make A produce a, or a produce A. Thus, hydrogen and oxygen, when fired, produce water; water galvanised, produces, hydrogen and oxygen. In this case the problem of finding a cause resolves itself into the much easier one of finding an effect, a problem to the solution of which experiment, and, therefore, direct Induction, is especially applicable.

With the exception of "*Transformations*," the investigation of complex effects by direct Induction is practised at such great disadvantages as generally to be impracticable. Our resource in such cases is *the Deductive Method*. This is especially true of the first division of complex effects, viz.,—*Compound Effects;* and in them, this inapplicability of direct Induction (observation and experiment) is, *cæt. par.*, in direct proportion to the number of the causes which conjointly produce the intermixed or complex effect, and to the smallness of the share which any one of these causes has in producing that effect. Mill proceeds to prove this in detail (see II.)

II. *The investigation of a complex effect may be conducted either:—*

1. *Deductively*—by *computing à priori* what would be the effect of the conjoint causes.

2. *By Induction*, either by way of :—
   (*a*) *Simple observation*—simply collecting instances of the effect as they occur ;
   (*b*) *Experiment* — *making* instances — taking the supposed causes and trying what effects they produce when conjoined.

Now Mill goes on to show that neither by (*a*) nor (*b*) can we effect much in investigating complex effects.

1. *Method of simple observation inapplicable.*

Take this example—"recovery from consumption"—a complex effect ; is the "taking cod-liver oil" one of its causes ?

It is obvious that many separate causes must combine to produce our effect ; now in such a case, where *many* causes are acting to one end, the share of each cause in the effect is not in general very great, and hence the effect is not likely to follow very closely any single cause in its presence or absence, still less in its variations.

2. *Method of Experiment is inapplicable, because we are unable to take certain precautions necessary to the scientific employment of experiment.* These are :—

(*a*) No unknown circumstances must exist in our cases. For instance, we should know everything (which can influence consumption) which exists in the system, when we give the oil.

(*b*) Known circumstances must not have effects liable to be confounded with the effect of the cause we are studying.

The most, then, we can hope to obtain by direct Induction in complex effects is, that a given cause is *very often* followed by a given effect.

To sum up then :—

Of complex effects, Transformations are the best adapted to inquiry by direct Induction ; next, the remainder of Heteropathic Effects ; and least of all Compound Effects, in proportion as the conjoined causes are numerous, and as each has but small share in producing the total complex effect. In all such cases of the inapplicability of Induction, the *Deductive Method* is our grand resource.

III. *Laws of Causation must be expressed as tendencies only.*

Every law of causation is liable *to be counteracted*, and apparently frustrated, by coming into contact with other laws, the results of which are more or less opposed to its result. Hence, with many such laws, cases in which they are entirely fulfilled do not, at first sight, seem instances of their operation at all. Suppose a ball to receive simultaneously in two exactly opposite directions two equal impulses, either of which would carry it a hundred feet in its own direction, the ball would, of course, remain unmoved, and no result would appear to follow, yet, in reality, each force fulfils its own law, and the ball occupies the same position as it would have done if the forces had acted successively instead of simultaneously. We must, therefore, define force as that which *tends* to cause motion in a body; and so in every other case of causation ; for though a cause always *tends* to produce its effect, counteracting causes may prevent that effect being manifested in the usual form.

IV. ***There is no such thing as a real exception to a general truth.***

The notion that there may be arises from neglecting the proper mode of expressing a law, as just explained. What is called an exception to a general principle is always a case of some other law interfering with it, and disguising or destroying its effect.

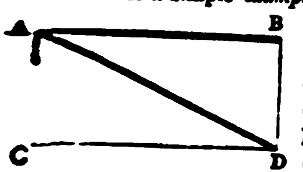
Doesn't it all depend on wording of genl. truth? – the insertion of condl. premiss &c.?

CHAPTER XI.

The Deductive Method.

I. The Deductive Method considers separately the causes which enter into the Complex Effect, and computes or calculates that effect, *à priori*, from the balance or product of the effects of the different causes which produce it.

Its problem, in fact, is to find the law of a Complex Effect from the laws of the different causes of which it is the joint result.



To take a simple *example*:—Suppose a particle at A is simultaneously acted upon by two forces, one of which, acting alone, would carry it to B in one second, and the other, in like manner, would carry it to C. Here the two forces are two causes uniting to produce a complex effect—

motion along the diagonal to D in one second. If, however, we suppose ourselves ignorant of what the result would be, it is evident we might discover it either by making numerous experiments with two forces, which would always give us a similar motion along the diagonal,—this would be *direct Induction*; or we might calculate *à priori* what the joint effect of the two causes or forces must be: we might argue, for instance, that the particle must evidently travel a distance equal to A B to the right of A, and a distance of A C below A; and the point D is the only point which fulfils these conditions, ∴ the particle at the end of one second must be at D, and so on;—this would be to apply the Deductive Method.

II. The Deductive Method consists of three distinct operations or steps:—

1. Ascertaining the laws of the separate causes by direct Induction.
2. Ratiocination from the Simple laws to the Complex case,—*i.e.*, calculating from the laws of the causes, what effect a given combination of them must produce.
3. Verification by specific experience.

1. ***It is first necessary to ascertain the laws of the separate causes.***

This is generally done by direct Induction, but if any of the separate laws be themselves complex, they may have been obtained by a previous Deduction. But even then, such complex laws being *ultimately* derived from simple or elementary laws (which are

always established by direct Induction), must *ultimately* be based upon inductions.

To this Induction it is essential :—

(1.) To know what the causes actually are whose effects we are about to study.

(2.) Their laws must then be ascertained.

This can only ultimately be done by the four Inductive Methods. And since the accuracy of this Induction is the foundation of the whole inquiry, it is necessary that, (*a*) if possible, we must study each of the concurrent causes in a separate state; for (*b*) if this is not possible—if we cannot try the effect of each cause apart from others, as in Biology—we experiment under great disadvantages.

2. *Ratiocination is the second step*,—i.e., *calculating from the known laws of the separate causes what effect any given combination of them will produce.*

On this we may observe :—

(1.) When our knowledge of the laws of the causes extends to the exact numerical relations which they observe in producing their effects, the ratiocination or calculation may reckon amongst its premisses all the theorems of the science of Number. Thus, we have a planet in its orbit round the sun at any moment under the influence of two separate causes—the central force of gravity pulling it towards the sun, and the tangential force. The actual path which it traces is the complex or joint effect of these two causes. Now it is clear that if we not only know the laws of these separate causes, but are able to state them with numerical exactness (as that

the central force varies inversely as the square of the distance and directly as the masses), any of the properties of squares, square-roots, &c., which we previously know, may be available in the calculation of the actual orbit.

(2.) When the effect takes place *in space*, and involves motion and extension, the theorems of geometry as well as of number come in as premisses. This is the case, for example, in Mechanics, Optics, Acoustics, and Astronomy. In the parallelogram of forces or velocities we are able to make use of any of the properties of parallelograms, triangles, &c., which we think fit in our calculation.

It might here, however, be naturally asked—How are we to be assured of the correctness of our calculation? How can we *know* that we have taken *all* the causes of our complex effect into account and rightly computed their joint results? To this we reply that we cannot have this necessary assurance of complete accuracy until we apply the proper *test*—verification by experience. Without this, deductive calculation is often nothing more than guesswork.

3. *Verification by comparison with the results of actual experience is the third step in an inquiry by the Deductive Method.*

Our calculations having led us to conclude that the effect will be of a certain kind, we must determine by actual observation or experiment whether it really is so or not. Having, from a knowledge of the laws of the central and tangential forces, calculated that a planet will move in an ellipse, we must verify the result by observing whether its successive places are really points on such a curve.

Two particular cases or forms of Verification may be noticed.

(1.) When the theory thus derived leads deductively to previously known *empirical laws* of the phenomena in question.

Thus Kepler's three laws were known as empirical laws—*i.e.*, as results of actual observation—before the time of Newton, who showed, however, that they were deducible from, *i.e.*, were results of, his theory of gravitation.

This is the most effectual verification possible.

(2.) When the theory is found to be in accordance with *a complex or obscure case.*

Thus the general law—"Heat is developed by compression of air"—was found to explain the observed fact that the calculated velocity of sound was less than the actual velocity. Now though we could hardly have discovered the law in question from this complex and obscure manifestation of it, yet when it was found that it exactly accounted for the difference observed, an important verification was supplied thereby.

[III. The Deductive Method presents several forms, according (1.) To the subject matter to which it is applied; and (2.) According to the mode of its application.

1. (*a.*) *The Abstract Deductive Method*, which deals with the laws of those sciences which are not concerned with causation, and therefore which are not liable to counteraction,—the laws of number and extension for example. Euclid's Geometry is an instance of the abstract or geometrical method.

(*b.*) *Concrete Deductive Method* deals with those sciences which are concerned with phenomena of causation.

2. (*c.*) *Direct Deductive Method*—in which we obtain our conclusion or law by Deduction first (*i.e.*, by a calculation of the effects of the conjoint causes), and afterwards verify by comparison with the results of experience.

(*d.*) *Inverse Deductive Method*—in which we obtain our law more or less conjecturally by direct experience, and afterwards verify it by showing that it is deducible from more general or better known laws.

In the Direct Method we compare the result of a calculation with experience; in the Inverse we compare experience with the result of a calculation.]

which that law is itself but a result, and from which it may be deductively inferred.

[*Popular and Philosophic Explanation.*

We must remember that all laws of nature are equally mysterious; we can no more assign a *why* for the more general than for the more special laws. But popularly an explanation means the substitution of a mystery which has become familiar, and so ceased to seem mysterious, for one to which we are still unaccustomed. An explanation in the philosophical sense, meaning merely the resolution of a law into more general laws, often does precisely the reverse of this,—it resolves a phenomenon with which we are familiar into one of which we previously knew little or nothing; as, for instance, when the familiar law, "All bodies tend to fall to the earth," was subsumed into or found to be of the previously unknown

1. *The first* is the case of the composition of causes, producing a joint effect equal to the sum of the separate effects. The explanation of such an effect evidently involves two things:—(1.) the simpler laws of the separate causes; and (2.) the fact of the co-existence of those causes (for if not co-existent they could not intermix their effects).

   Thus, in explaining the Compound Effect,—the orbit of a planet,—we must not only show that and how it results from the laws of the simpler causes, gravity and the tangential force, but also that those causes are actually conjoined, do really act on the planet.

   In this case the one law is resolved into two or more laws, all of which are more general and more certain than that law. (See p. 140.)

2. *The second mode* of explaining a law is to point out an intermediate link between an effect and its assigned cause, to show that this assigned cause is only the cause of the cause. *A* is supposed to cause *C*, but it is found that *B* is really the cause of *C*, and *A* is only the cause of *B*.

   In this case, too, the one law (*A* causes *C*) is resolved into two or more laws (*A* is cause of *B*, *B* is cause of *C*), each more general and more certain than the original law.

3. *The third mode* is the subsumption of less general laws into a more general one; that is, the less general laws are found to be merely instances of the operation of the more general.

   Thus the law that bodies fall to the earth was found to be a case of the great law of gravitation—the laws of magnetism a case of the laws of electric currents, &c.

   In this case two or more laws are resolved into

one, which law is evidently more general than the laws gathered up into it; but as to certainty no difference exists, since the less general laws are in fact the very same as the more general, and any exception to them would be an exception to it also.

III. Laws are always resolved into laws more general than themselves.

A law is said to be more general than another law when it extends to all the cases which that other extends to, and to others in addition.

This is self-evident in the third case. In the first and second, we find that the concurrence of two or more laws is required to give the less general law; thus, the law "*A* is followed by *B*," and the law "*B* is followed by *C*," are clearly more general than "*A* is followed by *C*," because, for instance, "*A* is followed by *B*" is fulfilled not only when *B* also produces *C*, but also in all other cases where the tendency of *B* to produce *C* is in any way counteracted. And, besides in the first case, the less general is fulfilled only in the cases where the simpler laws are co-existent in the required manner, while separately these simpler laws are fulfilled in many cases where the condition is wanting in addition.

IV. In the first and second cases a law is resolved into laws which are more certain than itself; in the third there is no difference in this respect.

Laws are said to be *certain* in proportion as they are less liable to exception,—*i.e.*, less liable to be

counteracted. It is perfectly clear that where a law is compounded of several others, the chance of its being counteracted is very much greater than that of any of the simpler laws which compose it. Each of these separately has only its own chances of counteraction, but the complex law has the sum of the chances of all. The chances of failure somewhere in a chain is very much greater than the chance of failure in any one particular link.

CHAPTER XIV.

LIMITS TO EXPLANATION OF LAWS OF NATURE.

I. We may recognise two kinds of Laws or Uniformities in Nature:—

1. Ultimate Laws.
2. Derivative Laws.

Ultimate Laws are those which cannot be resolved into (or deduced from or explained by) other and more general laws in any of the three modes of explanation just noticed.

This must be understood in a sense similar to that in which chemists speak of an "element," *i.e.*, something which cannot, by any *known* means, be resolved into simpler constituents.

Derivative Laws—those which can be thus resolved into other and more general laws.

Now, out of the total number of supposed Ultimate Laws, Science is continually removing some by reducing them to the class of Derivative Laws,—that is, showing that they are mere results of wider principles; and it becomes an interesting question how far this process may go on—to what extent may we expect to reduce the number of real Ultimate Laws; what indication have we as to the probable number of fundamental and Ultimate Laws, which being given, all other uniformities in Nature would follow.

II. The Ultimate Laws of Nature cannot possibly be less numerous than the distinguishable sensations or other feelings of our nature, —that is, those feelings which are distinguishable in *kind* or *quality*, and not merely in degree.

To explain—I am conscious of a certain sensation called a *sensation of red*, and also at times I am conscious of a sensation called a *sensation of sound*. Now these being phenomena which have a beginning, must have each its *immediate* cause, some antecedent which is invariably and unconditionally followed by a sensation of red, and some other followed similarly by a sensation of sound. Call the former cause *A*, and the latter *B*; now sound being a sensation different in kind from red, the law in virtue of which *A* is followed by a sensation of red, must always be distinct from the law by which *B* is followed by the mental state known as a sensation, or feeling, of sound. The one law can never be resolved into the other, they must always remain distinct Ultimate Laws; and so of every other case of sensations distinct in kind,—each must have its own ultimate law. From this it follows that—

III. The ideal limit of the explanation of natural phenomena would be to show that each distinguishable variety of our feelings has only one sort of cause.

That is, to show that, whenever, for instance, I am conscious of a red colour, that particular kind of sensation has always the same immediate cause, or antecedent, that our *A* is the same in every instance; and so of each distinct sensation.

IV. In what cases, then, has Science been most successful in explaining phenomena,—that is, in showing that supposed Ultimate Laws are really derivative?

Chiefly in the case of *motion*, for these reasons:—(1.) That phenomenon is always the same to our sensations in every respect, except as regards degree (for in the case where there is the greatest semblance of difference, motion in a straight line and curvilinear motion, the latter is only motion continually changing its direction); and (2.) it is a phenomenon which has an immense plurality of remote causes,—mechanical force, chemical, vital, electrical action, &c.

Now there is no absurdity in supposing that in every case motion may have the same *immediate* cause, or antecedent; and if such be the case, we may expect to bring the cases of motion produced

by some of these remote causes, under the same principle which operates when others of these remote causes give rise to motion. Accordingly the greatest achievements of science have consisted in doing this, as when the law of the fall of heavy bodies to the earth was found to come under the principle of gravitation, when magnetic movements were resolved into those produced by voltaic currents, &c.

CHAPTER XIV.—(*continued*).

On Hypotheses.

I. A Hypothesis is any supposition which we make, with avowedly insufficient evidence, in order to endeavour to deduce from it conclusions in accordance with facts which are known to be real.

II. *The purpose for which Hypotheses are framed is either :—*

1. The discovery by anticipation of a law of nature,—the hope being that the hypothesis is a correct statement of the real law. Now the only way to assure ourselves of this is to make inferences from the hypothesis, and by comparison of the results

with actual facts to prove or disprove our supposition. It is for this reason that Mill lays down that a "*legitimate*" or "*genuinely scientific hypothesis*" must be a *verifiable hypothesis*, — one, in its own nature, capable of being proved or disproved,—one destined not always to remain a hypothesis, but either be converted into a proved law of nature or abandoned as an error.

2. To fulfil certain *subordinate* but indispensable functions,—chiefly (*a*) to suggest new lines of investigation, and (*b*) to enable us to link together and form and retain a clear conception of facts already known.

It is not meant that any hypothesis, perhaps, is framed for either purpose exclusively, but that in most one or the other predominates. Many suppositions which can never perhaps be proved or disproved, as the atomic theory and the electrical theory of magnetism, have been in the highest degree serviceable as furnishing suggestions, and a clear order for the facts known.

III. There are two classes of Hypotheses :—

In the *first* the cause (if it be a case of causation) is real, but the law of its action is assumed ; in the *second* we assume a cause which is supposed to act according to known laws.

1. *Hypotheses of the first class—forms of :—*

(*a.*) Where, in a case of causation, we assume a law for a known actual cause,—*i.e.*, a cause not merely existing in nature somewhere, but known to have some actual nfluence on the given effect ; or at least to

Note

be one of a limited number, some of which are known to have such influence.

(b.) Where the hypothesis relates not to causation at all, but to the law of correspondence between facts which accompany each other in their variations.

For example, the hypotheses as to the law of the variation of the inclination of the refracted ray as the incident ray varies its angle of incidence, before the true law was known.

(c.) Hypothetical descriptions—that is, all suppositional modes of merely describing phenomena.

Thus, when we speak of "sun-rising," "setting," &c., these are merely suppositional modes of describing the phenomena visible. "It is as if so and so were the case" is the formula, and all that is necessary in any particular case is that this statement should be true.

In all these cases, verification is proof; the hypothesis may be received as true, merely because it explains the phenomena, since any hypothesis different from the true must lead to false results.

2. Hypotheses of the second class include those in which we assume a cause of whose connexion with the given effect we are not certain, or even, perhaps, of its actual existence in Nature at all.

In such cases we cannot have the assurance that a false law cannot lead to true results.

IV. Conditions under which a hypothesis may be received as true :—

1. We may dismiss those of the *second* class by remarking that the condition in them is that they should be reducible to the first. It is indispensable that the actual existence of the assumed cause, and its connexion with the effect, should be capable of being proved, and by evidence other than that of the facts which it is adduced to explain. Such a cause was what Newton meant by a *verâ causâ*. We say other and independent evidence, because a hypothesis of this class cannot be received as true merely because it explains all the known phenomena, for where we are at liberty to feign a cause, there is hardly any limit to the possible suppositions which will do this. Dr Whewell is wrong in laying down that such a hypothesis is to be received as true merely because it explains the phenomena already known, or even because its anticipations turn out to correspond with fact.

2. Of a hypothesis of first-class the conditions are :—

(a.) It must lead deductively to true results.

(b.) The case must be such that a false law cannot possibly give the true results.

Both of these are included in this one canon—that the final step, the verification, shall amount to, and fulfil the conditions of, a rigid Induction.

Such an Induction falls into the formula of the Method of Difference.

V. Subordinate functions of hypotheses.

It must not be assumed from what has been said that it is never allowable to imagine a cause; all that has been laid down is that such a supposition must not be received as true, merely because it explains the phenomena. The subordinate functions,—that of suggesting new lines of inquiry, and of affording a clear and connected view of known facts, and which

are *absolutely indispensable*, are often as effectually fulfilled as by hypotheses which we put forth as demonstrably representing the true law or fact.

VI. Some inquiries which deal with foregone collocations of causes are not hypothetical but inductive.

There is a great difference between inventing laws of nature to account for phenomena and merely endeavouring to conjecture what *collocation* now gone by may, in conformity with known laws, have given rise to facts now in existence. The latter is the strictly legitimate operation of inferring from an observed effect the existence in time past of causes similar to those by which we know the effect always now to be produced.

Thus, we have before us a certain effect, say the arrangement of certain geological strata; we know what causes and collocations would produce such an arrangement now, and from this we endeavour to infer what causes and collocations might have really formerly produced the effect in question.

CHAPTER XV.

On Progressive Effects.

I. A Progressive Effect is a complex effect, arising from the operation of one cause, by the continual addition of an effect to itself.

Thus, the fall of heavy bodies to the earth, sixteen feet in first second, forty-eight in second, and so, from action of one cause, gravity; continuous rusting of iron exposed to moist air, are progressive effects.

II. There is an obvious distinction between *temporary* and *permanent effects.*

Temporary, like a flash of lightning or explosion of gunpowder; *permanent*, those effects which remain unless some cause interfere to alter or destroy them.

Now, an agent or cause producing a permanent effect may, instead of being merely temporary, be itself permanent. In this case whatever effect has been produced up to a given time would subsist permanently (absence of altering causes being supposed), even if the cause were then to perish. Since, however, the cause does not perish, being permanent, but continues to exist and operate, it continues to produce more and more of the effect, and thus we get a *progressive effect*, from the accumulating influence of a single permanent cause.

III. This peculiar case is evidently only a case of the composition of causes,—the cause being here compounded with itself.

IV. There are two cases or kinds of Progressive Effects :—

1. When the cause though constantly acting is not variable.

2. When the constantly-acting cause itself varies.

In the second case, where the cause itself is variable, it is clear that it may be regularly or irregularly so, and if the former, it may be simply progressive or pass through a cycle of changes. In such cases the effect is progressive, as in the former case, but not *regularly* progressive; the quantities added to the effect in equal times are not equal.

V. How Progressive Effects are logically investigated.

We have already seen that cases of composition of causes can seldom satisfactorily be investigated, except by the Deductive Method; and this is pre-eminently true in the case of Progressive Effects, since the continuance of the cause influences the effect only by adding to its quantity; and since this addition takes place in accordance with a fixed law, the result can be computed mathematically,—the most complete example of the Deductive Method.

VI. Most uniformities of succession, which are not cases of causation (*i.e.*, a series of two or more terms in which each term is *not* caused by its predecessor) are cases of Progressive Effects.

In all cases of Progressive Effects there will evidently be a uniformity of succession between any stage of the effect and the next succeeding; thus, the second

particle of rust on iron succeeds the first, the third the second, and so on. And, generally, whenever we find any phenomena going through a regular process of variation, we do not presume that any term of the series is the effect of its predecessor, but rather that the entire series originates from the continued action of some permanent cause,—that, in a word, it is a Progressive Effect.

CHAPTER XVI.

Empirical Laws.

I. An Empirical Law is an observed uniformity, presumed to be resolvable into simpler laws, but not yet resolved into them; or it is a law whose *why* has not been ascertained.

The distinction between Ultimate and Derivative Laws has been already explained. Empirical Laws belong to the class of Derivative Laws, and constitute that section of them which has not been resolved into any simpler laws; thus—

Derivative Laws.	Resolved.	
	Not resolved.	Known to be cases of causation.
		Not known to be cases of causation.

We have thus two kinds or classes of Empirical Laws, and although all unresolved Derivative Laws may be termed Empirical, yet the designation, in *its strictest sense*, belongs to those not known to be cases of causation. For example,—let the Derivative Law assert that *A* is followed by *a*; if it belong to first class of Empirical Laws, we should know that *A* is the cause of *a* (but not why it is the cause); in the second class we should not know even that much.

II. Derivative Laws mostly depend upon collocations.

Since, when a Derivative Law is resolved it is usually found to be derived or to result from two or more simpler laws, it is evident that the fulfilment of the Derivative Law not only depends upon the existence of the simpler laws, but also upon their existence *together*, so as to intermix their effects. That is, the simpler agencies must be collocated, or arranged, in a particular manner.

III. Collocations of causes cannot be reduced to any law.

We cannot, in general, lay down any fixed principles applicable to the mode of arrangement or distribution of natural agencies.

IV. From this it arises that Empirical Laws cannot be relied on beyond the limits of actual experience.

This property is highly characteristic of Empirical Laws.

A Derivative Law which is resolvable wholly into a *single* more general law will be as universally true as that one law; but when it depends upon *several* such laws, we have seen that those laws must co-exist in a certain manner, or the Derivative Law will not be fulfilled. Now it is the very nature of a Derivative Law which has not been resolved, that we do not know whether it results from one law, or more than one; and if the latter, what collocations are necessary. Hence, we cannot be sure that the unresolved law will be found true, beyond the limits within which it has actually been found true, and where the necessary conditions are known to prevail.

V. Generalisations which rest solely on the Method of Agreement can only be received as Empirical Laws.

As already explained, we can never prove *causation* by this method; all that it proves is, that *two phenomena* (*A and a*) *are found together*,—an Empirical Law; we can never be sure that some unknown antecedent (*B*) is not either the cause of both phenomena, or causes one (*a*) while having been in our experience invariably conjoined with the other (*A*.)

Thus, suppose we have two phenomena,—redness of sky in the morning (*A*), and a fall of rain (*a*), these phenomena may be related in the following ways:—

A—a *A* being real cause of *a*.

$B<^{A}_{a}$ {*A* and *a* both produced by some unknown cause, *B*.

A
B—a {Some unknown cause, *B*, produces *a*, and has been always conjoined with *A*.

We may, however, add that in proportion as we have reason to suspect that the Empirical Law does not depend upon collocations, so may we rely more confidently upon it, in extending it to new cases. And also the more general an Empirical Law is, so is it the more certain.

VI. Signs by which an unresolved law may be presumed to be derivative and not ultimate.

1. Indications of any intermediate link between the antecedent and consequent.

If the effect is of such a kind that we see that it is probable that the obvious cause is not the immediate cause, we may infer that the law which connects such an effect with its obvious cause is resolvable.

2. Complexity of the antecedent.

In such a case we may infer that more than one of the elements which constitute the antecedent are concerned in producing the effect; and that therefore such effect is produced by conjunction of causes, and the law of its production is therefore resolvable into the simpler laws of the causes which concur in generating it.

CHAPTER XVII.

ELIMINATION OF CHANCE.

I. PROOF of Empirical Laws depends partly on Theory of Chance.

The method of agreement *per se* can only show that two phenomena are *conjoined;* and owing to plurality of causes, it does not prove that these conjunctions are anything more than mere unconnected coincidences, until the process for the elimination of chance has been applied.

As an example of a circumstance which is always coincident with phenomena on the earth's surface, and yet has nothing to do with many of these,—take gravity, or the magnetic influence of the earth.

II. Definition of Chance.

Chance only applies to conjunctions (sequences or co-existences) of phenomena; and conjunctions of phenomena are said to happen casually or by chance, when those phenomena are in no way related through causation.

No phenomenon, or event, can properly be said to be *produced* by chance,—*i.e.*, immediately produced

Given the cause or combination of antecedents, and the effect necessarily follows. But when an event is said to be produced by chance, what is really meant is, that the *conjunction* of antecedents or conditions upon which that phenomenon followed, happened without any causal connexion between them.

Take this case,—the particular position of Jones in a battle, and the particular path of a bullet in that battle, killing Jones by shooting him through the heart. Now Jones' death was an effect necessarily consequent upon his heart being in the path of the bullet: so far the effect followed its cause without any question of chance: but the conjunction of Jones' particular position and the particular path of the ball might be wholly casual.

The Elimination of Chance is applied in two distinct cases :—

1. In the case of conjunctions of phenomena (as just explained).
2. In the case of a constant cause associated with casual causes; and this again presents two cases, as we shall see.

III. Application of this process to conjunctions of phenomena.

The question we have to solve is,—After how many and what sort of instances may it be concluded that an observed coincidence (conjunction) between two phenomena is not the effect of chance?

This question is answered through another,—Do the

coincidences occur more (or less) frequently than chance will account for? Are the two phenomena more or less frequently conjoined than would occur on the supposition that no connexion exists between them?

No general answer can be given to this inquiry; all we can do here is to point out *the principle upon which we must proceed in eliminating Chance in cases of conjunctions of phenomena:*—

We consider first the absolute frequency of each phenomenon itself; and then how great frequency of conjunction must therefore be expected without supposing any connexion (either of tendency to cause or of tendency to prevent the one or the other) between them. If we actually find them conjoined more frequently than this, we may (with certain precautions which we will pass over for the present) infer that there is *some* connexion; if less often, that there is some repugnance.

Thus, suppose we are considering whether there is any connexion between a red sky and rain, we should first determine the absolute frequency of each phenomenon. Suppose we find that a red sky occurs one day in three, and rain one day in two, in every six days there would be a casual conjunction between them, and half the number of red skies would be in that way associated with rain. This, then, is the frequency of conjunction due to chance, with which we have to compare the observed frequency of conjunction.

IV. Application of the process in the case

of a constant cause associated with casual causes.

As an example of this case,—the position of the sun is the constant (*i.e.*, for a few days) cause of the temperature of any given day, but with the effect of this are blended the effects of many casual causes—wind, clouds, &c.

There are two cases of this combination :—

1. Where the effect of the constant cause forms so great and conspicuous a part of the total result that its existence as a cause could never be a matter of uncertainty.

Here the elimination of chance determines how much of the total effect is due to the constant and casual causes respectively.

2. Where the effect of the constant cause is so small compared with that of the changeable or casual causes, that its very existence as a cause may be unknown.

Here the process actually determines the existence of the constant cause as a cause.

Case 1. *Where effect of constant cause is large compared with total effects of casual causes.*

Suppose a chemist to weigh repeatedly the same body, the actual result which he obtains in any one experiment is mainly determined by the true weight of the body (this is the conspicuous constant cause A); but, besides this, the result would be modified in each case by *casual causes*, — sources of error, — draughts, imperfections of the balance, &c., which would vary in each experiment. Now if

(*a.*) We could completely isolate the constant cause A (*i.e.*, in this case remove all sources of error), we could, of course, directly determine the part of any given effect due to it; but if not,

(*b.*) We must apply the process of eliminating the effects of the casual causes in the following manner :—

We make as many trials as possible, A being preserved invariable; the results of these different trials will naturally differ, but (if, as we here assume, the casual causes in the long run tend as much for as against the constant cause) in such a manner as to oscillate about a certain point, sometimes being greater, sometimes less. If so, we may conclude that that mean or average result is the part due to our constant cause A; the variable remainder being the effect of the casual causes.

The test of the sufficiency of this Induction is, that any increase in the number of trials does not materially alter the average.

Case 2. Where the effect of the constant cause is comparatively inconspicuous—in such a case we may *discover a residual phenomena (i.e., constant cause) by the elimination of chance.*

This is merely a particular case of the process just described, but here the very existence of the constant cause is determined by the same method which, in the previous case, served to ascertain the quantity of its effect.

This case of Induction may be characterised thus:—A given effect is known to be chiefly, and not known not to be wholly, determined by casual causes. If,

indeed, it be wholly so produced, then the average of the effect will be zero, the effects of the casual causes cancelling each other. If, however, the average be not zero, but some positive quantity, about which the total effect oscillates equally,—sometimes above, sometimes below,—we may conclude that this quantity is the effect of some constant cause, which cause we may set about detecting.

Thus, a slightly loaded die may be detected in this way; the loading is a slight permanent cause, mixed with casual causes. If the die were fair, in a large number of trials we should get about the same number for each face, but if there is a steady preponderance in any particular number, we may be sure that the die is proportionally loaded.

V. We know, if we make a *very large number* of throws with a fair die, that each of the six faces of the die will come uppermost about the same number of times; but if we make only a limited number of trials, there may be considerable deviation from this *average*; for instance, in four trials, the six may come up twice or even three times. In like manner, in reference to the coincidences of phenomena which we are now discussing, besides the question,

What is the number of coincidences which on an average of a great number of trials (or in the long run) may be expected to arise from chance alone? which has been considered in this chapter, there is another:—

Of what extent of deviation from that average is the occurrence credible from chance alone in some limited number of instances? This question is answered by the doctrine of Calculation of Chances, which is next considered. (See Chap. XVIII., Subdivision V.)

CHAPTER XVIII.

Calculation of Chances; or, Theory of Probability.

I. The probability of a given event to any one is the degree of expectation of its occurrence which he is warranted in entertaining by present evidence.

It is very necessary to remember that the probability of an event is not a quality of the event itself, but merely a name for the degree of ground which some individual has for expecting its occurrence. Every event is *in itself* certain to occur or not to occur, and if we were omniscient, there would be no such thing as probability.

There are two forms of Probability:—

1. Simple probability without specific knowledge (*i.e.*, of the comparative frequency with which the different possible events in fact occur).
2. Probability based more or less upon such specific knowledge.

II. The first form of Probability is characterised thus,—"Out of several events we must know that some *one* will certainly happen, and

one only; and we must not know, nor have *any* reason to expect, that it will be one of these rather than another."

Suppose we are required to take one ball from a box, of which we only know that it contains black, white, and red balls, and none of any other colour. Here we know that either a white, or black, or red ball will be drawn, but we have no reason for expecting one rather than the other. In such a case, the drawing of any particular colour, is equally probable to us, and it will be indifferent upon which we stake; while, if we stake *against* any colour, the odds are two to one against us. This is an example of the first kind of probability; but if we have some specific knowledge of the actual frequency with which white, black, and red balls were drawn, if, for instance, several trials had previously been made, or if we knew that there were more black than white or red balls, it would become a case of the second kind,—that is, *we should have some reason to expect one event rather than the other.*

The *basis* of this first kind of probability is clearly the general and axiomatic principle, that out of the possible cases there must be a majority against each, except one at most; and since we cannot presume any one to be in this position, we have no ground for electing one rather than another. This principle being universal, there is, in our reasoning, no reference to *specific* knowledge,—knowledge special to the particular case.

Although the first kind of Probability is interesting as a mathematical study, yet the practical probability with which Logic and non-mathematical sciences are

concerned belongs almost exclusively to the second kind, and this for two reasons,—(1.) because, as regards the first kind, in practical questions it is usually impossible to make out the list of events which are possible; and (2.) because it is very rare that we cannot obtain some specific knowledge which will immensely aid us in our calculation.

III. In the second kind, therefore, with which we are here concerned, our conclusions respecting the probability of a particular fact rest upon more or less knowledge of the proportion between the cases in which it occurs and in which it does not occur; this knowledge being either derived from specific experiment, or by reasoning upon the causes in operation which tend to produce, compared with those which tend to prevent, the fact in question.

The *basis* then of our estimate of this kind of probability is either an *Induction* or a *Deduction*, and it is very necessary to notice that it is not unimportant which. If on trial we find that we draw one black ball on an average to every nine white ones, the conclusion that an event occurs once in ten times is as much an induction as that the event occurs uniformly. But when we arrive at a conclusion in this way by simply counting instances in actual experience, we only get an empirical law, we know nothing of the "*why.*" But if we know *the causes* which operate for or against, we can *deduce* the pro-

bability in a much more certain manner. Indeed, it is really by a Deduction that we infer that from an urn containing ten black and ninety white balls we may nine times as much expect to draw a white ball as a black, because we consider that the hand may alight in nine places and find a white, and in only one and find a black. So a betting man does not rest content with the results of actual trials of the horses, but examines the animals separately, to get as much as possible at the causes of superior speed. This, too, is the reason why the first occurrence of an event, about the possibility of which there might be a doubt, adds so much more to its probability that any subsequent happening of it does, because we are thereby assured that causes really exist adequate to produce it, which till it actually occurred we might not be certain of.

Still, however, as a matter of fact, in most cases in which the estimation of chances is applied to practical uses on a large scale, the data are drawn not from knowledge of the causes, but from direct experience of the relative frequency of events of the different kinds. The chances of life, of recovery from disease, of accident, shipwreck, &c., are drawn from registers of the actual occurrence of those phenomena,—*i.e.*, from the observed frequency not of the *causes* but of the *effects* themselves.

IV. Theory of probability which relates to the cause of a given event (effect).

The question is—Which of several causes is most likely to have produced a given effect?

Answer.—Given an effect to be accounted for, and there

being several causes which might have produced it, but of the presence of which in the particular case nothing is known; the probability that the event was produced by a given one of these causes is as the antecedent probability of the existence of that cause, multiplied by the probability that that cause, if it existed, would have produced that effect.

V. Given the *average* number of coincidences to be looked for between two unconnected phenomena,—What are the chances of any given deviation from that average? (See end of last chapter.)

We observed a certain number or succession of coincidences between A and B; if A and B are merely coincident by chance, we know how many such coincidences ought to be expected; the question then follows, what are the chances that the *observed* coincidences between A and B are merely casual? If *not* casual, they must be the result of some law.

To answer this we compare the two probabilities:—(1.) the probability that the coincidences are due to chance, and (2.) the probability they are due to some law.

As to (1.), if the known probability of a single coincidence be $\frac{1}{m}$, the probability that the same coincidence will occur n times in succession is $\frac{1}{m^n}$.

As to (2.), the probability that the coincidences are due to some law, admits of more or less exact estimation according to circumstances:—

(*a.*) We may know what the cause of the conjunc-

tion must be if a cause exist at all, and we may be able to estimate the probability of its presence.

(b.) But if we do not know any known cause which would account for it, it is clear we cannot estimate the probability of the presence of an entirely unknown thing, and we are driven to the result of the first question,—are the conjunctions so successive or numerous that their being produced by chance would be a very uncommon thing? If so, since it is not a very uncommon thing to discover a new cause in nature, we may conclude that such does here operate in causing the conjunctions, and may lay down provisionally an empirical law.

CHAPTER XIX.

Extension of Derivative Laws to Adjacent Cases.

I. Derivative Laws are less general and less certain than the Ultimate Laws into which they are resolvable, or presumed to be resolvable, for two classes of reasons:—

(1.) They very often depend upon collocations (co-existence in a particular manner) of causes for which there is no law.

(2.) They are more apt to be counteracted; and for one

phenomenon to be produced independently of the other.

This has already been explained of the different forms of derivative laws (see page 140), and we only remark again here, that every derivative law which is resolvable into or derived from two or more simpler laws, will only be fulfilled where the two simpler laws are found together,—*i.e.*, its fulfilment depends upon collocations. Thus the rise of mercury in the Torricellian tube is a derivative law, resolvable into two simpler laws, gravity and equal transmission of liquid pressure. Now if these two laws do not act together (as when the Torricellian experiment is made in a vacuum), the derivative law (that the mercury will rise) is no longer fulfilled. So also the immense majority of derivative laws depend partly upon collocations.

Again, when we have two effects conjoined, arising from the same cause, say *a* and *b* from the cause *c*, it may happen that either *a* or *b* may be produced *alone* by some other cause than *c*, and then in that case *a* and *b* will not be found together, and the law which asserts that *a* and *b* are always conjoined will fail.

II. From all this it follows that Derivative Laws (and especially unresolved Derivative Laws) must only be extended to cases adjacent (*i.e.*, contiguous or similar) in Time, Place, and Circumstances.

1. As regards *Time*, take the derivative law that day and night succeed each other.

Now we extend this law with confidence to cases

adjacent in time,—we expect that day and night will succeed each other for some time to come. In this case we know the causes,—the derivative law has been resolved,—they are the opaque earth rotating upon an axis, lying in a certain direction, and the sun shining. Now, as long as these causes exist, and are not counteracted, the derivative law will not fail; but we know by observation that these phenomena have continued unaltered for at least five thousand years, and, therefore, during that time no counteracting cause has diminished them in any appreciable degree. And it is opposed to all experience that such a cause should start into immensity in a single day, or in a short time.

If we did not know the causes of day and night, we could still draw a similar conclusion with less extension as to the future. We should know at least that the phenomena had been for five thousand years conjoined, and we might therefore infer that the causes had not been counteracted during that period, when the same conclusions would follow as in preceding case. But still we could not be *assured* that if we did know the causes we could not predict their destruction from agencies actually in existence. Thus, a clock has been made to go for years without interference; a savage wholly ignorant of its mechanism, seeing it going on steadily day after day, might easily imagine its movement perpetual, but a person who knows the causes, knows that it contains within itself the causes of its own cessation.

In either case the argument becomes weaker in proportion as we extend the period which our prediction covers.

2. As regards *Place*, it might seem that an empirical law could not even be extended to adjacent cases, for the existence of a cause in any one place is no guarantee of its existence in any other place. When, therefore, we extend such a law to cases beyond the limits of place in which it has been observed, such cases must be presumably within the influence of the same individual agents.

CHAPTER XX.

On Analogy.

[It is very necessary to clearly understand and keep in view the different senses in which the term "Analogy" may be used in different cases.

The general formula of inference from experience may be given thus:—

A certain object (or set of objects) *A*, having a certain property (or set of properties) *x*, have also a certain property *m*;

Another object *B*, resembles *A* in possessing *x*;

∴ *B* also resembles *A* in possessing *m*.

If for *A* we substitute "certain animals," for *x* "split hoofs," for *m* "rumination," and for *B* "some particular animal," the form of the argument will be seen.

Now, this general formula includes several particular forms of reasoning:—

1. Analogy in the strict sense.
2. Example.
3. Imperfect Induction.
4. Perfect Induction.

1. *Analogy* in the strict sense, or resemblance of relations.

2. *Example.*—The characteristic of this is, that between x (the properties which constitute the resemblance between A and B), and m (the property inferred to exist in B), no connexion whatever is known, nor are they known to be not connected.

Thus, let A be "the earth;" x spherical shape, rotation on axis and atmosphere; m being inhabited, and B Venus, then the argument of Example stands thus:—

The earth possesses spherical shape, &c., and is also inhabited.

Venus resembles the earth in possessing the former set of properties.

∴ Venus is inhabited.

Now, in this case, no connexion whatever is known to exist between the properties which form the resemblance and the inferred property, "being inhabited," and we do not *know* that they are unconnected with it. This form, therefore, is the argument from *simple or mere resemblance.*

3. *In Imperfect Inductions some* connexion is shown between the resembling properties and the inferred property, short of the former being a mark of the latter.

4. *In Perfect Inductions* we show, by a due comparison of instances, that x is a mark of m, that the properties which constitute the resemblance are invariably accompanied by the inferred property. This constitutes *the distinction between "Analogy" and Perfect Induction.*

Mill does not make use of the term "*Example*," but, after discussing "*Analogy Proper*," he goes on to consider what is here called "*Example*," under the general head of Analogy; and this plan we shall follow here to avoid deviating from his phraseology.]

I. Analogy in the strict sense.

Here the two resembling things (our A and B) are *relations*, as we might say, the relation between a despot and his people resembles the relation between a father and his family.

The condition of the validity of this form of argument is this,—that the two relations should really resemble each other in that particular fact or circumstance upon which the inferred property depends.

II. Analogy in general.

The argument from Analogy amounts to this,—a property, m, known to belong to A, is more likely to belong to B, if B agrees with A in some of its properties (though we know no connexion between m and any of these properties), than if no resemblance whatever could be traced between B and A.

It is requisite to an analogical argument of this kind—first, that the property, *m* (being inhabited), shall not be known to be connected with any of the common properties of *A* and *B* (the earth and Venus); and, secondly, *m* must not be known to be unconnected with any of these common properties, or such clearly count for nothing in the argument.

Every additional resemblance (in points not known to be unconnected with *m*) between *A* and *B*, so far favours the conclusion that *B* possesses *m* like *A*.

Every additional dissimilarity, in like manner, between *A* and *B* weakens by so much the conclusion.

The value of an analogical argument, then, depends upon three things:—

1. The amount of ascertained resemblance between *A* and *B*.
2. The amount of ascertained dissimilarity between them.
3. The amount of unascertained properties in *A* and *B*.

Such an argument may, therefore, come very near to a valid Induction, if—

1. The resemblance is very great.
2. The dissimilarity very small.
3. Our knowledge of the subject-matter tolerably extensive.

Suggestive use of Analogy—

In this respect analogical considerations have often the greatest scientific value; there is no Analogy, however faint, which may not be of great importance in

suggesting experiments or observations which may lead to more positive conclusions.

[III. *Imperfect Inductions.*

Here we show that *some* connexion, short of actual causation (which would be a Perfect Induction) exists between the properties which constitute the resemblance and the inferred property, *m*. We may show that the former is an important part of the cause of the latter, or that it has a tendency to prevent the existence or effects of counteracting causes, &c. &c.]

CHAPTER XXI.

EVIDENCE OF THE LAW OF CAUSATION.

I. The Inductive Methods presuppose the universality of the Law of Causation; if we believe that any phenomenon can start into being without a cause, the conclusion from any one of them fails at once.

Since, then, this great law lies at the bottom of our Inductive inferences, upon what evidence is it itself based?

II. Answer of the Intuitive School.

They say that the truth of the law is certain because the mind cannot help believing it; it is an intuitive truth, acquiescence in which is necessitated by the laws of the thinking faculty.

Mill replies thus :—

In opposition to this view I must reiterate my protest against adducing as evidence of the truth of a fact of external nature any necessity the human mind may be supposed to be under of believing such fact. It has already been shown (page 77, &c.) that such conceivability or inconceivability of anything depends mainly upon the mental history of the person making the attempt. Moreover, in this case, the statement is not true; there is no difficulty in conceiving that phenomena may start into being without a cause, for many philosophers believe that the impulses of the will are of this character, and the ancients recognised "chance" and "spontaneity" amongst natural agencies.

III. The Law of Causation is really proved by an Induction *per enumerationem simplicem*.

As men recognised one instance of causation after another, they would first suspect that many effects have a cause; as experience widened, that most events have; and finally, as an immense number of experiences accumulated, without any certain exception, they would believe that the law was universal, perhaps really before, in logical strictness, they were warranted in so doing.

IV. Under what conditions Induction *per enumerationem simplicem*, is a valid process.

We have already laid down that this method is valid, precisely *in proportion to our assurance that if an exception ever did occur we should know of it.* In other words,

Precisely as the subject-matter is limited and special, so is the process insufficient and delusive.

As its sphere widens, this unscientific method becomes less and less liable to mislead; and the widest truths, viz., the law of Causation, and the primary laws of number and extension, are proved by this method alone, nor are they susceptible of any other proof.

Induction by simple enumeration leads to Empirical laws; these cannot be extended beyond adjacent cases, because the causes may cease to exist or be counteracted, or the requisite collocations may fail. If, however, we suppose the subject-matter of our law to be so widely diffused that there is no time, place, or set of circumstances in which it is not fulfilled, it is clear that the law cannot be frustrated by any counteracting causes except such as never occur, and cannot depend upon any collocations except such as exist at all times and places. It is therefore an Empirical law co-extensive with human experience, at which point the distinction between Empirical laws and laws of nature vanishes, and the proposition takes its place amongst the highest order of truths accessible to science. Such is the Law of Causation.

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V. The Law of Cause and Effect being thus certain, is able to confer the same certainty upon all laws which can be deduced from it.

The utmost we can do in the way of proof for any law is to show that it is true, or the Law of Causation is false,—that a phenomenon has the assigned cause or none. The Inductive Methods are formulæ of the modes of doing this.

VI. Summary of proofs which we *now* have of the universal and absolute truth of the Law of Causation.

1. We *know* it to be true of by far the greater number of phenomena.
2. There are none of which we know it not to be true.
3. Of those phenomena of which we do not positively know it to be true,
   (a.) One after another is constantly passing from this class into that of known examples of its truth, as they are better understood.
   (b.) And the deficiency of our positive knowledge with respect to these phenomena may always be accounted for by their rarity or obscurity.
   (c.) And finally, every such phenomenon, produced apparently without cause, obeys in some other respects known laws of nature, and therefore we may presume this law also.

We may, however, add that these reasons do not hold for the prevalence of this law beyond the limits of our experience, in distant stellar worlds for example.

CHAPTER XXII.

Coexistences independent of Causation.

I. Properties of objects may be divided into two classes :—

1. Those which are results of Causation (derivative).
2. Those which are ultimate,—which are not results of Causation, which are not connected with antecedent phenomena by any law.

Thus, if we contemplate the substance "oxygen," we find it to present numerous properties; some of them are manifestly derivative, depending upon assignable causes; thus, its gaseous form is probably due to latent heat; but after subtracting all such, there remains a number which seem inherent in oxygen, which are themselves the causes of other properties, but are not themselves caused by any, these are ultimate properties.

It must not be supposed that we are always able to determine whether a given property is really ultimate or not; very often we know positively that it is derivative, and in no case can we be certain that it is not. Still, for practical purposes, we regard those properties as ultimate, which we have no reason to suppose derivative.

II. The coexistences, then, with which we are now concerned, are the coexistences of

these ultimate properties; A and B are found together, but their coexistence cannot be accounted for by Laws of Causation,—we know no reason *why* they should, but only that they do coexist.

III. Mode in which propositions assertive of coexistence of ultimate properties are proved.

Propositions expressive of such coexistences are to be regarded as Empirical laws, and the only proof of which they are susceptible is by the Induction by simple enumeration.

The reason why we are thrown back upon this method is, because there is no general axiom bearing the same relation to uniformities of coexistence as the law of Causation bears to uniformities of succession. If B follows A, and we can show that A is the cause of B, then we may be sure that where A is present (counteracting causes being absent) B will also be found,—A will be a mark of B. But if A and B are found to coexist, and their coexistence cannot be traced to causation, we have no similar axiom to give us an assurance that they will be invariably coexistent,—that where A is, B also will always be found. The overlooking of this distinction was the grand error in Bacon's system of philosophy.

IV. Empirical Laws are stronger in proportion as they are more general.

The condition of the validity of the method of Induction by simple enumeration has already been several times pointed out; as its sphere widens it becomes more and more trustworthy, and in the last chapter it has been shown that there is a point of generality at which Empirical laws become as certain as laws of Nature,—or rather there is no longer any distinction between them.

For (1.) if an Empirical law be really a law of Causation, the more general it is, the greater is proved to be the space over which the necessary collocations prevail, and within which counteracting causes do not exist; and (2.) even if not a result of Causation, but expressive of an ultimate coexistence (as we here are specially considering), the more general the law is, the greater amount of experience it is derived from, and *the greater the probability therefore that if exceptions ever occurred, we should know of them.*

Hence it requires stronger evidence to establish an exception to a law of this kind, in proportion to its generality.

CHAPTER XXIII.

Approximate Generalisations and Probable Evidence.

(An Approximate Generalisation is one of the form "Most A is B.")

I. When a conclusion is said to rest upon *probable evidence*, the premisses from which it

is drawn are usually approximate generalisations.

As every certain inference implies that there is ground for a proposition of the form "All *A* is *B*," so every similar probable inference implies that there is ground for an assertion of the form "Most *A* is *B*;" and the degree of probability in such a case will depend upon the proportion between the number of instances which agree with, and the number which conflict with, the generalisation.

II. Approximate generalisations are of much less value for the purposes of Science than for the purposes of practical life.

Why of inferior value in Science?

Because, beside the inferior precision of such propositions, and beside the inferior assurance with which they can be applied to particular cases, they are almost useless as a means of discovering ulterior truths by way of Deduction. In a Syllogism which contains an approximate generalisation as a premiss, we can only at the very utmost obtain another approximate generalisation as a conclusion, and generally no conclusion at all.

Their use in Science, then, is chiefly as suggestions of, and materials for, universal truths.

Why of greater value as practical guides?

Because (1.) they are often our only resource; (2.) the laws of phenomena, even when known, are commonly too much encumbered with conditions to be adapted for every-day use; and (3.) our decision is often required so rapidly that we are compelled to act upon probability, without waiting for certainty.

The principles of Induction, therefore, applicable to approximate generalisations are not a less important subject of inquiry than the rules for arriving at universal truths, but they are mere corollaries from these latter.

III. There are two classes of cases where we are forced to be guided by approximate generalisations:—

1. When we have nothing better; when we have not been able to carry our investigations of the laws of the phenomena any further.

As an example we may give Newton's generalisation—Most highly refractive substances are combustible.

The importance of this class is not very great.

2. When the generalisation (Most *A* is *B*) is not the ultimatum of our scientific knowledge, but the knowledge we really possess beyond it cannot be conveniently brought to bear upon the particular case.

In such cases we really know what circumstances distinguish those *A*'s which possess *B* from those which do not, but we have not the time or means to examine whether such distinguishing circumstances exist in the present case or not.

This is usually the case with inquiries of the kind called "moral;" *i.e.*, having for their object the prediction of human actions. Nearly all propositions relating

to these must be thrown into the form—"Most persons act so and so in such and such circumstances." Not but that in general we know well enough upon what internal dispositions and external circumstances the actual result will depend, but we have seldom the means of knowing whether and how far any given individual possesses those internal qualities, or is under the influence of those external circumstances.

We may, therefore, divide Approximate Generalisations into two Classes:—

1. Where they embody the total or ultimatum of our knowledge.
2. Where they do not (but constitute the most available form for practical guidance).

IV. How approximate generalisations are proved:—

It is necessary to consider separately the two forms of these laws:—

1. Where the approximate generalisation in question comprises all we know of the subject.

We know only that "Most A is B;" not why they are so; nor in what respects those which are, differ from those which are not.

In this case we arrive at "Most A is B," in precisely the same manner as at "All A is B," if the latter happened to be the truth in the case. We collect a number of instances sufficient to eliminate chance conjunctions of the phenomena A and B,

and having done so, we compare the number of cases where A possesses B with the number of cases where it does not, and frame our conclusion accordingly.

2. Where the approximate generalisation is not the ultimatum of our knowledge.

When we know not only that "Most A is B," but also the causes of B, or some properties by which the portion of A possessing B is distinguishable from the portion of A without B (criteria of B).

In this case we have a double mode of proving that "Most A is B;" (1.) the *Direct*, as in the preceding case; and (2.) the *Deductive* or Indirect,—examining whether the proposition can be deduced from the known causes or known criteria of B.

[Example:—Suppose the proposition to be proved is "Most Scotchmen can read;" by the *direct* mode we should examine a sufficient number of Scotchmen, and deduce our conclusion accordingly; by the *indirect*, knowing that a cause of "being able to read" is "being sent to school," and a criterion of it would be the actual reading of a newspaper, we might inquire what proportion of Scotchmen were sent to school, or to what extent newspapers, &c., were circulated amongst them.]

It is evident that in cases of this class we may always substitute a universal proposition for the approximate, by introducing the cause or criterion as a qualification. Thus—"All Scotchmen, who have been properly taught, can read."

V. Precautions in arguing from approximate generalisations to individual cases.

1. As to the direct application of an approximate generalisation to a single instance.

Approximate generalisations, being Empirical laws, can only be relied upon as regards cases presumably within the limits of time, place, and circumstances of our observation.

If the proposition "Most A is B" has been sufficiently established as an Empirical law, we may conclude that any average A is B, with a probability proportionate to the preponderance of affirmative over negative instances in our experience, care being taken that our A is a fair average instance.

2. Application of two or more approximate generalisations to the same case. This may occur in two ways:—

(*a.*) By addition of probabilities,—the "Self-corroborative chain" of Bentham. The type is this—"Most A is B," "Most C is B," this case is both A and C, ∴ it is probably B.

Thus, Most of *M's* assertions are true (two in three); Most of *N's* assertions are true (three in four); this assertion is both *M's* and *N's*,—what is the chance that the assertion is true in which they both thus concur? If we arrange twelve statements of each, one under the other, we shall see that they both together speak truth six times in twelve, and both false together once in twelve,—therefore, if they both agree in an assertion, it will be true six times for once it is false, or the chances of its truth are $\frac{6}{7}$.

(*b.*) By Deduction,—"Self-infirmative chain" of Bentham. The type is "Most A is B," "Most C is A," this is a C, ∴ it is probably an A, ∴ it is probably a B.

Here the degree of probability of the inference decreases at every step, and even where two probabilities only are concerned, may amount almost to nothing. But if the probabilities are taken fairly in reference to each other, we may say that the probability arising from the two propositions taken in this way together is equal to the probability arising from the one abated in the ratio of that arising from the other.

Thus—Most inhabitants of Stockholm are Swedes (eight out of nine); most Swedes have light hair (nine out of ten); ∴ probability that any given inhabitant of Stockholm is light haired is $\frac{8}{10}$ ($=\frac{8}{9}\times\frac{9}{10}$).

The first form (*a.*) is exemplified when we prove a fact by the testimony of two or more independent witnesses; the second (*b.*), when we adduce the testimony of one witness that he has heard the assertion from another.

VI. There are two cases in which reasonings, depending upon approximate generalisations, may be carried as far, and are as strictly valid, as where based upon universals.

This is an example of the current saying, "An exception which proves the rule;" approximate generalisations are capable of being made use of in this way, because they can be turned into general propositions in form or in fact.

1. *First Case.* When the approximate generalisations are of the second kind—*i.e.*, when we are cognisant of the character which distinguishes the cases which accord with the generalisation from those which are exceptions to it. In such cases we can, if we choose, substitute a universal proposition with a proviso,

for the approximate generalisation; and however many steps are involved in a train of reasoning, the successive provisos being carried forward to the conclusion will exactly indicate how far that conclusion may be positively relied on.

Thus, take these approximate assertions,—"Most persons possessing uncontrolled power employ it ill," and "Most absolute monarchs possess uncontrolled power," ∴ "Most absolute monarchs employ their power ill;" we may change them respectively into universals thus—"All persons not of unusual strength of mind and confirmed virtue possessing, &c.;" "All absolute monarchs not needing the active support of their subjects possess, &c.;" ∴ "All absolute monarchs not of unusual strength of mind, &c., and not needing, &c., employ their power ill."

Second Case is where the inquiry relates not to the properties of individuals, but of multitudes; since what is true approximately of the individuals, is true absolutely of the mass.

For example—"Most Englishmen are Protestants" is equivalent to "The English people are a Protestant people."

This is chiefly exemplified in the groups of Social Sciences; and hence we see the error in the common opinion that speculations on Society and Government, as resting on apparently mere probable evidence, must be of inferior certainty and reliability.

CHAPTER XXIV.

REMAINING LAWS OF NATURE.

We have already seen (pp. 30 and 31) that every Real Proposition must assert one of the following:—

Existence,
Order in Time,
Order in Place,
Resemblance.

Up to this point we have considered the Logic of *Propositions which assert Order in Time* (including *Causation* and the other forms of *Sequence* and *Co-existence in Time*), the modes in which they are proved, and the conditions of their validity. It now remains to do the same for each of the other three great classes into which assertions may be divided.

I. Propositions asserting Simple Existence.

a. The existence of a phenomenon is but another word for its being perceived, or for the inferred possibility of perceiving it by somebody or other.

It is true that a thing is often said to exist when it is absent or past, and is not and cannot be perceived;

but even then its existence is to us only another word for our conviction that we *should* perceive it upon a certain supposition,—if we were placed in the needful circumstances of time and place, and endowed with the necessary perfection of sensory organs.

b. Existence of anything is established either—

1. By *immediate observation*, when the thing is within the range of immediate observation; or,
2. By *inferring its existence through marks or evidences*, when it is beyond that range; *i.e.*, from other phenomena known by Induction to be connected with the phenomenon by way of succession or coexistence. In such cases we prove the existence of a thing by proving that it follows, precedes, or coexists with some known thing, and this last is accomplished by ordinary Inductive processes.

c. General propositions affirming Simple Existence are sufficiently proved by a single instance.

That "ghosts" or "sea serpents" exist would be positively established if it could be shown that such things had even once been really seen.

[It should have been remarked as regards the ordinary Inductive methods, proving coexistence or sequence, that *two instances at least are necessarily required, though these two instances may be given by a single experiment;* and it is this that is really meant when we speak of a single instance being sufficient to support such a generalisation. If we refer to the canon of the Method of Difference (which applies to such cases) we shall see that two *instances* are necessary,

—one of the presence, the other of the absence of the phenomenon. It is only, therefore, generalisations asserting *existence*, which are really established by a single *instance*.]

II. Propositions asserting Resemblance (or the contrary).

(Resemblance and its opposite include likeness, similarly; and of number or magnitude—equality, inequality, similarity, proportionality.)

Resemblance between two things may be established in two ways:—

1. *Immediately*—by direct comparison of the two things.
2. *Mediately*—when each is compared with some third thing.

The comparison of two things through the intervention of some third thing, when their direct comparison is not practicable, is the appropriate scientific process for ascertaining Resemblances, and is the sum total of what Logic has to teach on the subject.

[Error of Locke and Condillac school.

This error consisted in the view that Reasoning itself was nothing but the comparison of two ideas through the medium of a third; and knowledge, the perception of the agreement or disagreement of two ideas.

This view of Reasoning and Knowledge requires to be restricted to our Reasonings about and our Knowledge of Resemblances, and not even then is the comparison made between our ideas of pheno-

mena but between the phenomena themselves. It is only in mathematics that the comparison is really made between the ideas of things, and it is so because our mental pictures of form, of lines, circles, &c., are as much fitted for comparison as pictures of them upon a surface before us,—being perfect transcripts, *quâ* form, of the realities.]

Propositions asserting Resemblance may be divided into two classes :—

1. Where the resemblance arises merely from a cause operating in a certain way. Thus, the angle of incidence equals the angle of reflection, asserts resemblance, but it is only in consequence of being the very law of the cause to act in that manner.
2. Resemblances true of all phenomena without reference to their origin. These are the Laws of Number and Extension, in other words of Mathematics.

The first class of resemblances are merely part of the laws of the production of the phenomena, and these are amenable to the principles of ordinary Induction. It is only the latter class which it is necessary to consider here, and we may therefore say that, *of propositions asserting Resemblance, the laws of Mathematics alone require a separate logical consideration.* A similar assertion we shall also find to be true of the next class of propositions, those asserting *Order in Place;* which will therefore be next dismissed, leaving only the Mathematical Sciences generally for final consideration.

III. Propositions asserting Order in Place

May be classed thus :—

1. Those asserting Order in Place of the effects of causes,

are mere corollaries of the laws of the causes, and resolved by ordinary inductive processes.

2. Those asserting the Order in Place or the collocation of the primeval causes. These assert in each instance an ultimate fact, in which no laws or uniformities are traceable.
3. The only remaining general propositions asserting Order in Place are some of the propositions of Geometry; laws through which we are able, from the situation (Order in Place) of certain points, lines, or areas, to infer that of others which are connected in some assigned manner with the former.

THE METHODS OF THE PURE MATHEMATICAL SCIENCES.

I. These are :—

Science of Number, including	Arithmetic, Algebra, The Calculus, &c.
Science of Extension or Space,	Geometry.

II. General remarks on the fundamental principles of Mathematics :—

1. These fundamental principles are (1) *Axioms,* and (2) *Definitions* so-called ; the last involving a postulate or implied assertion of the real existence of the thing corresponding to the name defined, which

postulate can alone form the basis or premiss of scientific Deduction.

2. These fundamental principles are based upon experience; being, in fact, proved by an Induction *per enumerationem simplicem*, and one of such wide generality as to be a perfectly rigorous proof.

3. Comparison of the ideas of things (as regards their *form*) is strictly equivalent to comparison of the things themselves, since our mental pictures of not too complex forms are perfect transcripts of the realities.

4. The following include the reasons why the primary truths of Mathematics seem to have a greater certainty than other inductive truths:—

a. *Their universality.*—They are true of everything, everywhere, and at every time.

b. *Their extreme familiarity.*—The perception of their truth only requires the simple act of looking at objects in the proper position, and often only thinking of them in such a position. Hence exemplifications of their truth are incessantly presented to us.

c. *The absence of any analogies to suggest a different law.*—This is very important; if everything in the universe always maintained a condition of absolute rest, we might find it as difficult to conceive the possibility of the sun falling from the sky as we now have of conceiving that two straight lines can enclose a space.

d. *They are never counteracted*, being independent of causes.

III. The Science of Number.

1. *Generally.*

a. The elementary or fundamental truths of this branch of Mathematics are:—

1. Two axioms— { Things equal to the same are equal to one another. If equals be added to equals the sums are equal.

2. Definitions— { Of the various numbers, &c., explaining a name and asserting a fact.

The other axioms may be deduced from these two.

b. Every name of a number (one, two, three, &c.) *denotes* physical phenomena, and *connotes* a physical property of those phenomena, and that is the characteristic manner in which the agglomeration or whole is made up and may be separated into its parts; in other words, the manner in which objects must be put together to form that number. Thus the name "two" connotes an impression on the senses similar to that made by . .; "three" by . . . or .˙., and so on; the higher the number the more the ways in which it may be made up.

c. *All the theorems of the Science of Number assert the identity of different modes of formation.* They assert that some mode of formation from x, and some mode of formation from a given function of x, produce the same result.

d. *The general problem of the Science of Number is*—Given a function, what function is it of some other function? F ($=3a^2$) being a certain function of a given

number (a), what function will F be of any function of that given number (a)?

(A function of x being any expression which contains x.)

2. *Arithmetic.*

a. Every arithmetical proposition, every statement of the result of an arithmetical operation, is the statement of one of the modes of the formation of a given number.

Suppose we take the number 1728,—this may be considered as made up in an infinity of ways; thus, as $1000 + 700 + 20 + 8$, &c. &c. To say $12^3 = 1728$ is to affirm that one way of making up 1728 is to cube the number 12. When one mode of forming a number is given we can find any others which are required.

b. *What renders Arithmetic a Deductive Science* is the applicability to it of the comprehensive law—"The sums of equals are equals," or "Whatever is made up of parts is made up of the parts of those parts." Every arithmetical operation is an application of this law or some law deducible from it.

3. *Algebra.*

a. The propositions of Algebra affirm the equivalence of different modes of formation of numbers generally; they are true not only of particular numbers, but of all numbers.

It is true that different numbers cannot be formed in the same way from the same numbers, but they may be formed in the same way from different numbers; and this way algebraical propositions assert.

b. *Algebraical Notation* is a system of nomenclature devised for the purpose of enabling us to carry on general reasoning about functions, by enabling us to express any numbers by names which, without specifying what particular numbers they are, shall show what function each is of the other; in other words, shall show the mode of formation from one another.

c. *The general problem of Algebra is—F* being a certain function of a given number, to find what function *F* will be of any function of that given number; or, given a function, what function is it of some other function?

IV. Science of Extension (Geometry).

a. The elementary principles of this branch of science are, as before explained, *Axioms* and *Definitions*, with their contained postulates.

b. Every theorem of Geometry is a law of External Nature; and this would have been perceived (or that Geometry is a strictly physical science), in all ages, had it not been for the illusion produced by two causes.

(1.) The characteristic property of the fundamental facts, that they may be collected from our ideas or mental pictures of the objects as effectually as from the objects themselves.

(2.) The demonstrative character of geometrical truths; *i.e.*, certain suppositions or hypotheses being granted, Geometry deduces from these what conclusions it can.

c. Why Geometry is so eminently deductive.

(1.) All questions of position and figure can be resolved

into questions of magnitude;* and thus Geometry is reduced to the single problem of the ascertainment of the relations of quantity (chiefly equality, sometimes proportionality) between magnitudes; and by the aid of the axioms relating to equality, each equality becomes a mark of an almost infinite number of others.

(2.) Three of the principal laws of extension or space are unusually fitted for rendering one position or magnitude a mark of another, and thus making the science deductive. These are:—(*a.*) Magnitudes of enclosed spaces are determined by magnitudes of enclosing sides and angles; (*b.*) the length of any line may be measured by the angle it subtends, and *vice versâ;* and (*c.*) the angle which two straight lines make with each other at an inaccessible point is measured by the angle which they severally make with any straight line we choose to select.

V. Function of Mathematics in other sciences.

Causes, like everything else which can have quantity or position, operate *pro tanto* under mathematical laws; and in proportion as any science affords precise numerical data, so is the applicability of mathematical principles to that science.

VI. Limits of that function.

(1.) When causes are so imperfectly accessible, that we cannot get their numerical laws.

* Thus the position and figure of any line, plane, or solid, is determined if we know the position of a sufficient number of points in it; and the position of any point may be fixed by the *magnitudes* of two or three co-ordinates drawn in reference to two or three axes.

(2.) Where they are so numerous or complexly intermixed that the calculation would transcend human powers.

(3.) Where the causes are perpetually fluctuating, as in Biology and Sociology.

The value of Mathematics as a preparatory study is chiefly as presenting the most perfect type of the Deductive Method.

CHAPTER XXV.

On the Grounds of Disbelief.

By Disbelief is here meant not mere doubt, but positive disbelief; so that even if evidence of apparently great strength is, or may be, produced for the proposition, we consider the witnesses in error, or the evidence somehow wrong.

The positive evidence produced in favour of an assertion that is thus disbelieved is always grounded on some *approximate generalisation;* such as—"Most things asserted by a number of respectable witnesses are true;" or "Most of the impressions made on the senses accord with reality." The affirmative evidence, then, being never more than approximate generalisation, the whole question depends upon what the evidence *against* the proposition is. If this is an approximate generalisation, it is a case of comparison of probabilities; but if the evidence be a higher truth, we are guided by the following:—

I. Canon of Disbelief.

If an assertion be in contradiction (not

merely to any number of approximate generalisations, but) to a complete generalisation, grounded on a rigorous induction,—*the absence of adequate counteracting causes being supposed*,—it is impossible, and is to be disbelieved totally.

An induction of the kind referred to may be obtained when the Inductive Methods capable of giving rigorous results have been rigorously carried out. If, indeed, we find it necessary to admit the inconsistent assertion, we must give up the law, as somehow involving a mistake.

II. Two cases may be considered under this Canon:—

1. Where the assertion appears to conflict with a real law of Causation.

Facts contrary to experience (Hume). Facts disconformable *in toto* or *in genere* (Bentham).

This is probable or improbable in exact proportion to the probability or improbability that there existed in the case in question an adequate counteracting cause.

2. Where the assertion conflicts with mere uniformities of coexistence, not proved to be dependent on Causation.

Facts unconformable to experience (Hume). Disconformably *in specie* (Bentham).

It is with these uniformities principally that the marvellous stories of travellers conflict. What is really asserted in cases of this nature is the existence of a new kind. This is not by itself at all incredible, and having fairly considered the probability that the alleged new kind could have escaped previous observers, the assertion is to be tested by the principle before laid down,—that it is improbable in proportion to the generality of the uniformity with which it conflicts.

[III. Hume's doctrine of Miracles.

Hume, as is well known, laid down an argument against miracles in this form,—"It is improbable that a miracle should be true; it is improbable that the testimony in its favour should be false; we have therefore a comparison of improbabilities; but the former improbability is the highest possible, being an impossibility, for 'anything which is contrary to experience is impossible.'"

The last proposition is the important part of the argument, and it is, in fact, nothing more than a loose statement of the Canon of Disbelief. But on referring to that Canon, we find that the incredible assertion must not be merely that a cause existed without being followed by its effect, but that this happened in the absence of counteracting causes.

Now the assertion in the case of an alleged miracle is the exact opposite of this,—it is, that the effect was defeated, not *in the absence*, but *in consequence*, of a counteracting cause; that cause being the direct action of supernatural power. If such a cause exist in the case, there is no question of its competency;

the only improbability is whether it really did operate in the given instance.

All, therefore, that Hume has made out is this—that no evidence can prove a miracle to any one who does not previously believe in the existence of a superior being, or who thinks that such being would not interfere. Nor, again, is it possible for the miracle itself to prove the existence of supernatural agency, for there is always the alternative *possibility* of some unknown natural cause.]

IV. The absolutely impossible assertions

Are such as contradict the Laws of Number or Extension, or the universal law of Causation. An assertion at variance with either of these is absolutely and for ever incredible,—in other words, impossible.

Summary.—We have thus far, then, considered assertions conflicting

(1.) With approximate generalisations—we must balance probabilities.

(2.) With laws of causation—probability depends on probability of presence of counteracting cause.

(3.) With uniformities of coexistence—upon probability of existence of the new kind.

(4.) With laws of Number, Extension, or Causation,—absolutely impossible.

And we now proceed to consider one or two minor points.

V. Improbability before the fact (*i.e.*, the chances being against a thing before it happens) must not be confounded with improbability after (difficulty of believing a thing said to have actually occurred).

Many events are altogether improbable to us *before* they have happened, or before we are informed of their happening, which are not in the least incredible when we are informed of them, because not contrary to any, even an approximate, generalisation. That a given individual will die in a particular manner, at a particular moment, twenty years previously is highly improbable, though we believe the statement directly we are assured it has happened on credible authority. Dr Campbell in his reply to Hume has confounded these two kinds of improbability.

VI. Probability of Coincidences.

Suppose a person assures us that he has seen the six thrown ten times in succession by a die, which we have by previous trials ascertained to be perfectly fair, what is the credibility of the assertion?

It is perfectly evident, in the first place, that a series of ten sixes is, *in itself*, just as probable as any other series of ten numbers,—if therefore we are inclined to discredit the assertion of the former, where we should believe the latter, it must be, *not because the former assertion is less likely to be true*, but *because it is more likely to be false.* Motives to falsehood, of

which one of the most frequent is the desire to astonish, are more likely to have operated in the wonderful assertion.

Such a regular series certainly *seems* more improbable than any given irregular series, but this is only because the comparison is tacitly made between it and all irregular series taken together. Of course if before the ten throws we were asked to guess whether the series would be regular or irregular, we should say irregular, but this is only because the possible irregular combinations are immensely more numerous; if we had to determine whether a *given* irregular or a *given* regular series were more probable, it would be quite indifferent on which we fixed.

BOOK IV.

OPERATIONS SUBSIDIARY TO INDUCTION.

CHAPTER I.

Observation and Description.

The "*operations subsidiary to Induction*," are *Observation*, *Abstraction*, *Naming*, and *Classification*. Observation is here considered; and the term in this place is understood as including Experiment, not as contrasted with it.

I. Observation takes its place first amongst operations subsidiary to Induction.

For Induction is the extension of something which has been *observed* to be true in certain members of a class to the whole class. As far as Logic is concerned (its function being the estimation of evidence), we have only to consider in reference to observation,—

II. What condition is necessary in order that any fact supposed to be observed, may safely be received as true?

The sole condition is that which is supposed to have been observed shall really have been observed; that it is an observation, not an inference.

We have already pointed out (p. 3), the large share occupied by rapid and unconscious inferences in what are ordinarily supposed to be direct impressions on the senses. In strictness, observation only extends to the sensations themselves which we receive from objects, or to the mind's internal states; the very existence of the object is an inference from these sensations; and what Logic, therefore, has to teach on the point is this,—that, in such cases, we should be aware of what is really observed, and what is inferred, and should remember that an error *may* lurk in the latter process. Errors of sense so called, are, in fact, erroneous inferences from sense, the sensations themselves must be real.

III. A description of an observation affirms more than is contained in the observation; it is inherent in a description to be a statement of resemblance or resemblances.

To describe an object is to assign attributes to it, and in doing this we necessarily assert a resemblance between the object and everything else which possesses any of those attributes. If I say, "This object is white," I incidentally affirm a resemblance between this object and all other white objects.

This resemblance to some other objects may be ascertained by direct comparison, as in the instance just given, or *deductively* by marks of the attribute. Thus, when we say, "The earth is a sphere," we affirm resemblance to all other spherical bodies; but this is not obtained by direct comparison, but by marks of the spherical form in the case of the earth (these marks being circular horizon, disappearance of hull of ships first, &c).

CHAPTER II.

Abstraction, or the formation of Concepts.

I. A general notion or concept is the conception of a multitude of resembling individuals as an aggregate or class.

Such general notions certainly exist in the mind, *i.e.*, the mind can in some way or other conceive a multitude of individuals, as an assemblage or class; otherwise we could not use general names with any consciousness of meaning.

How such notions are obtained, and what is their precise nature, are questions with which Logic is not concerned; it is sufficient for its purpose that the name of a class calls up some idea by which we can think and speak of a class as such.

II. A general conception of the phenomena

we are investigating is a necessary preliminary to that comparison which Induction presupposes.

That is, we cannot ascertain general truths,—truths applicable to classes,—unless we first form the classes; and this is done by comparing individuals and ascertaining their common properties.

III. General conceptions do not develop themselves from within the mind, but are always, in the first instance, impressed upon it from without.

Dr Whewell holds that the mind is, as it were, a storehouse of conceptions of every kind, which are produced from it from time to time as they are found to suit particular aggregates of objects; the mind, as it were, produces conceptions as a tree fruit, without assistance from impressions from outward things. Mill replies, that general conceptions are always obtained by abstraction from individual objects, either—(1.) from the very things we are at the moment examining, of which we are endeavouring to ascertain the points of agreement; or (2.) from things which we have perceived or thought of on some former occasion, whose points of agreement we have already ascertained, and which we find to coincide more or less accurately with the points of agreement of the phenomena we are at the moment investigating. Thus, suppose I have placed before me a number of points placed along the circumference of a circle; if I have never seen a circle before, I may gather a conception of it from the very instance before me; if I have already the notion of a circle, I simply have to compare it with the present case, to find whether it agrees or not.

It is evidently only in the latter class of cases that the conception can be said to pre-exist in the mind; but even here the conception was originally obtained by observation and comparison of external phenomena.

IV. Type function of general conceptions.

A general conception itself, originally the result of comparison, becomes the type of comparison.

The human mind cannot properly compare more than two objects together simultaneously. Having, by this first comparison, noted the most prominent points of agreement, or apparent agreement, and having thus framed our first rough general notion, when another object, apparently of the same class, is presented to us, we naturally compare it, not with either of the two first, but with the aggregate of points in which they agreed,—in other words, with our first general conception. This original conception may thereby be found to require correction or to admit of extension; and so we go on, comparing our conception with one individual after another, till we get a sufficiently precise general idea of the class. In a word, it is perfectly clear that when we have to compare a large number of individuals together, a type of some kind is indispensable, and the general conception is the only really useful type.

V. Tentative process in forming or applying general conceptions.

If, after forming our first rough general conception, we proceed further to compare it with one individual after another, if it do not seem suitable it becomes necessary either to correct it or even abandon it altogether. In the latter case we must begin again, and look out for a fresh set of agreements, and form and try a new conception.

It is this tentative process which seems to have suggested Dr Whewell's view—that general conceptions are furnished by the mind and from the mind itself. In such cases, it is true, the conception is furnished *by* the mind, but not till it has first been furnished *to* the mind by the contemplation of outward phenomena.

VI. What is meant by appropriate conceptions.

Appropriate is an elliptical term, meaning appropriate to some particular purpose; and an "*appropriate conception*" is one which comprehends not only real points of agreement, but also such points of agreement as are important relatively to our particular purpose.

Thus, a gardener's conception of "seed," "fruit," or "flower," would be different from a botanist's, yet each would be most appropriate for its own purpose.

VII. What is meant by clear conceptions.

A conception is clear when we know exactly in what the agreement between the different phenomena consists; in other words when we know accurately

what simpler notions our general conception is made up of, or comprehends.

It is not necessary for a conception to be clear that it should be *complete*,—that is, that we should know *all* the common properties of the things we class together.

VIII. The *appropriateness* of our conceptions (or rather the chance of our hitting upon points of agreement appropriate to our particular purpose) depends chiefly upon the *activity* of our observing and comparing faculties.

The *clearness* of our conceptions depends chiefly upon the *carefulness* and *accuracy* with which we employ those faculties.

The *chief requisites for clear conceptions* are, therefore—(1.) Habits of attentive observation; (2.) Extensive experience; and (3.) A memory which receives an exact image of what is observed, and tenaciously retains it.

CHAPTER III.

NAMING, AS SUBSIDIARY TO INDUCTION.

PRELIMINARY remarks.

1. The remarks in this chapter relate to *general names*, that portion of language with which Logic is chiefly concerned.

2. The *logical use of names* is either :—

(1.) *Indirect*,—the same as their uses as instruments of thought.

(2.) *Direct*,—enabling us to lay down general propositions.

3. *The use of names as instruments of thought* consists—

(1.) In their power of binding up simpler ideas into convenient groups; and (2.) enabling us to produce these groups when wanted. In these ways they form a powerful artificial memory, and very much shorten thinking.

General names aggregate together attributes into such groups as are wanted for use; a name, for example, like "*civilisation*," binds up into one whole a number of simpler ideas—a certain state of intellectual cultivation, a certain condition of moral feeling, and of the knowledge of art and science, &c. If we had no such word as "civilisation," we should not only be compelled to enumerate all these every time we wanted the complex idea, but we should probably fail to remember them, since there would be nothing to throw them into a group and rivet them together. So also is it with other general names; they, in fact, perform in the mind the same function as the binding does to the books of a library; without such, the mind would resemble a library of books, all in separate leaves, confusedly mixed.

I. Names are not absolutely indispensable for inference.

We can make some inferences without the use of language, namely, simple cases of direct inference from particular cases to another particular case, without passing through a general proposition. If reasoning consists in recognising one phenomenon as a mark of another, nothing clearly is *absolutely* requisite, except *senses* to perceive that two phenomena are conjoined, and an *associative law*, by means of which one of these raises up the idea of the other.

II. That our inferences without language, however, would be very limited and precarious may be inferred when we remember that the direct use of names is to enable us to lay down and preserve general propositions; and that the uses of general propositions are :—

1. They enable us to avail ourselves of our past experience, by regularly registering it in assertions.
2. As also of the collective experience of mankind.

3. They enable us to permanently record or register uniformities.
4. They enable us, moreover, to do this once for all.

III. General names are not a mere contrivance for economising language.

Some have supposed that the necessity of general names springs from the immense multitude of individual objects, preventing us having a separate name for each of them of which we may wish to speak. This, if true to some extent as regards the use of general names in ordinary language, is not true of their logical use. Even if we had a separate proper name for each individual thing, without general names we could register neither the results of our comparisons nor uniformities.

Rigorously speaking, we could carry on logical operations without any other general names than the abstract name of attributes; our propositions having this form, "A possesses such and such an attribute;" or this—"Attribute A is conjoined with or resembles attribute B."

CHAPTER IV.

Principles of Definition.

The *chief requisites of a philosophical language* (*i.e.*, a language perfectly suitable for

the investigation and expression of general truths) are:—

1. That every general name employed should have a meaning precisely determined and steadily fixed.
2. That we should possess a name wherever one is needed; and that this name should fulfil certain ends in the best manner.

It is to the discussion of the first of these, or the Principles of Definition, that this chapter is limited.

I. To attach a certain and definite meaning to every general name, is the same thing as to assign to every general concrete name *a definite connotation;* for if the name be abstract, its meaning is at once settled by the connotation of the corresponding concrete.

II. Concrete general names are very often used with indefinite connotation; and this arises in two ways:—

1. Such words are used without connoting any distinct attribute at all, but merely a vague general resemblance to other things called by the same name.

   When ordinary people speak, for instance, of an action as "just" or "noble," they imply no distinct quality of the act, but merely that it resembles acts to which they have been accustomed to hear those terms applied.

2. The connotation derived from *accustomed predication*

often assists in giving a vague meaning to the name.

When a general name stands as the subject of a proposition, predicates or attributes are affirmed of the objects it stands for. Now, if a name is frequently employed as a subject, and a certain set of attributes are often predicated of it, these attributes often become mixed up in a vague way with the original meaning of the name. Just as from frequently hearing the ass called stubborn or foolish, we come to regard the name "ass" as connoting stubbornness or foolishness.

III. The framing of a satisfactory definition of a name in common use is not an arbitrary process; neither does it depend wholly upon such common usage, but also upon a knowledge of *the properties of the things* denoted.

For, in order to assign a connotation to a name, consistently with its continuing to denote certain objects, we have to make our selection from amongst those attributes in which the objects agree. The process involves two inquiries—In what do the objects agree? What attributes have they in common? And, having settled this, the second is—Which of these common attributes will serve best to mark out the class from all others, and ought therefore to be assigned as the connotation of the name?

In answer to this second inquiry, we say,—those attributes ought to be selected which are, as far as possible—

1. Sure marks of the greatest number of other important properties.
2. Such as are familiarly predicated of the objects.
3. Such as have the greatest share in producing the general resemblance amongst the objects.

IV. The transitive application of words.

1. *What is meant by it.*

When men meet with an object new to them, there is a strong tendency not to invent a new name, but to apply to the new object the name of some familiar object which seems to resemble the new one most. Thus the word "oil" (*oleum, oliva*) originally meant olive oil exclusively, but as new objects were continually being discovered bearing more or less resemblance to olive oil, the name oil was by degrees extended to a very large number of bodies,—to sulphuric acid, for example; and even to solids, as palm oil. Now a name, in this transitive way, may pass on from object to object, till at last the denotation becomes so wide that the various included objects have but little or anything in common.

2. *How the logician should deal with such cases.*

He should be careful not unadvisedly to discard any of the connotation of the name, but should, if it be necessary, rather restrict its denotation, by dropping some of the objects to which it has been extended.

3. *Important law of mind in reference to these transitions.*

When the word has passed naturally and easily from one

shade of meaning to another, the association between the different meanings may become virtually indissoluble, and the various transitive meanings will coalesce into one complex conception; the meanings will blend together in the mind, and the real transition becomes an apparent generalisation.

V. It is an important fact that there is a constant tendency in names to lose portions of their connotation, from habitual inattention to the total of ideas conveyed by the name; and this is especially likely to occur when the connotation is left vague and unsettled.

CHAPTER V.

History of Variations in Meanings of Terms.

I. Accidental connotation; collateral associations affecting words.

The incorporation into the meaning of a word of some circumstance originally accidental is a frequent cause of variation in meaning; and such accidental connotation may not only be incorporated into the word, but may in the end more or less completely supersede the original meaning.

A name which is in every one's mouth derives its con-

notation from the circumstances which are habitually brought to mind when it is pronounced; but if any circumstances happen to be so frequently associated with these as to be constantly suggested when the name is used, they may become as much part of the meaning of the name as those originally brought to mind by it. Thus, *pagan* originally meant "dweller in a village;" but since such persons were usually behind the age, ignorant and heathenish, these accidental circumstances gradually became incorporated with the meaning of the name, and at length formed its exclusive meaning.

This continual incorporation of meanings originally accidental is the reason why—

(1.) There are so few exact synonyms in a language; and

(2.) Why the dictionary meaning of a word is often so imperfect an exponent of its real meaning.

II. Transitive change in meaning of words has been already noticed. (P. 217.)

III. Alterations in the meaning of a term must evidently consist either in one of these two things, or of both together:—

1. Loss of some part of connotation (Generalisation).
2. Taking on of fresh connotation (Specialisation).

That is, a word must either come to mean more or to mean less, or to mean more in one direction while meaning less in another. Of course as connotation

is increased, denotation is diminished, and *vice versâ*.

1. *Generalisation.*

May happen :—

(*a.*) From dropping a part of connotation from mere ignorance of the omitted portion.

(*b.*) The fact that the number of known objects multiplies faster than names for them gives rise to a tendency to give to a new object the name of an old; and thus the name, extending its denotation, lessens its connotation.

Is most likely to occur to the greatest extent in words expressive of the complicated phenomena of mind and society.

2. *Specialisation.*

May happen :—

(*a.*) Words originally expressing a general character becoming limited to some particular object.

Thus Arsenicum meant originally any strongly irritant substance, but afterwards became limited to the substance now known by that name.

(*b.*) From the habit which persons, whose attention is frequently directed to certain species of a genus, have of giving the name of the genus to that species.

Thus, to a sportsman "bird" means a "partridge" or "grouse."

(*c.*) An idea sometimes becomes incorporated into the meaning of a word from mere chance conjunction. (See Accidental Connotation.)

(*d.*) From the common practice of using general terms where more specific words might be employed; thus the wider term gradually gets a specific connotation.

A practice has a tendency to grow up in a polite society of designating objects by the most general words which will suffice to point them out; and thus such words often pick up additional meanings.

The additional connotation which a word soonest and most readily takes up is that of agreeableness or disagreeableness in some of its forms; that a thing is good or bad, desirable or the reverse, and so on.

CHAPTER VI.

TERMINOLOGY AND NOMENCLATURE.

HAVING in Chapter IV. discussed the *first* of the two main requisites of a philosophical language, we now in this chapter proceed to the consideration of the *second* of these, namely :—

I. That we should possess a name wherever one is needed (*i.e.*, there should be a name for everything about which we have often to speak).

[For the subordinate clause—"this name should fulfil certain ends in the best manner," —see Chap. VII., p. 208.]

This second requisite involves three sub-requisites :—

1. An accurate descriptive Terminology.

2. A name for each important result of Scientific Abstraction.

3. A Nomenclature, or System of Names of Kinds.

1. *An accurate descriptive Terminology.*

A Terminology is a System of Terms or Names; and by a name being "accurately descriptive" is meant this—that it should be capable of conveying an exact notion of the phenomenon to another person, as do the words "hunger," "blackness," &c., in common speech.

Strictly speaking, a name for every variety of simple or elementary feeling would be sufficient; but it conduces much both to brevity and clearness to have separate names for oft-recurring combinations of feelings.

When a name is appropriated to a previously unnamed phenomenon, the new name ought to be associated immediately with the phenomenon or feeling to which it has been assigned, so as to recall it without delay or trouble.

2. *A name for every important result of Abstraction,—i.e.,* a name for every important common property, or aggregate of common properties, which we detect by comparison of the facts.

There are three advantages connected with the appropriation of a single definite name to the abstracted quality:—

(a.) Its use saves time, space, and circumlocution.

(b.) It promotes perspicuity by enabling us to reason with or about the conception *as a whole,* without being confused by thinking unnecessarily of its parts; just as mathematicians substitute a single symbol for a complex expression.

(c.) A name fixes our attention upon a phenomenon and causes it to be remembered.

3. *There must be a name for every Real Kind,—in other words, a Nomenclature.*

A Nomenclature may be defined as—A collection of the names of all the Real Kinds with which any branch of knowledge is conversant; or more strictly, Of all the lowest Kinds or Infimæ Species.

Such is exemplified in Botany, Zoology, and Chemistry: *Viola Odorata, Felis leo, Ferric oxide,* are examples from Systems of Nomenclature.

There is a peculiarity in the connotation of Names which form part of a Nomenclature,—namely, that besides their ordinary connotation, as concrete general names, they have a special one; besides connoting certain attributes, they also connote that those attributes are distinctive of a Real Kind. A definition can only express the former; and hence an appearance that the signification of such terms cannot be completely conveyed by a definition.

II. *A third and subordinate aphorism respecting a philosophical language* may be laid down thus:—

Whenever the reasoning can be carried on mechanically, without risk of error, the language should be rendered as mechanical as possible; but if not, every precaution should be taken against such a mode of

using it. In connection with this we proceed to notice a few points respecting—

III. Mechanical use of language.

1. *What is meant by it.*

The complete or extreme case is when language is used without any consciousness of meaning, and with only the consciousness of using certain visible or audible signs in conformity with technical rules previously laid down.

2. *When applicable and not.*

(*a*.) Mechanical use of language can never be permitted or be useful in our Inductive operations.

(*b*.) And in our Deductive only when our reasonings are independent of any property *peculiar to* the things with which we are concerned,—*i.e.*, only when we are concerned with properties which are properties of all things whatever.

(*c*.) Therefore, practically speaking, its use is limited to our reasonings about *Number*.

(*d*.) In all other sciences, then, except Number, we must endeavour as much as possible to *prevent* ourselves from using language mechanically; and this end is accomplished (1.) by throwing as much meaning as possible into words; and (2.) by frequently calling up the ideas involved in their meanings.

CHAPTER VII.

Classification as subsidiary to Induction.

There are two kinds of Classification (see also p. 37):—

1. That form which is inseparable from the use of general names. As has been already remarked, every name which connotes an attribute, incidentally divides all things into two classes,—those which possess, and those which do not possess, the attribute in question. Such a classification includes not only all things which are known or which exist, but all which may be imagined or hereafter be discovered.
2. In the other kind of classification, with which alone we are here concerned, the arrangement or distribution of things is not a mere incidental consequence, but the end and aim of the process; the naming being secondary to and in conformity with the classification. Such are the classifications of Botany, Zoology, &c.

The principles of Scientific Classification have reference to a twofold object:—

1. The arrangement of the objects of its study into *Natural Groups*, with the object of facilitating our inductive inquiries generally.
2. The arrangement of *Natural Groups into a Natural*

Series, with the object of facilitating some special inductive inquiry.

It must not be supposed that the arrangement into a Natural Series is merely a further stage of the arrangement into Natural Groups; the two are distinct both in their principle and their object. It is with the arrangement into Natural Groups that we are concerned in the present chapter; the "Classification by Series" is discussed in Chapter VIII.

I. "*Natural Groups*" are classes of such a kind that the things included therein resemble each other most in the general aggregate of their properties. Such groups of individuals, species, or genera, as would be spontaneously framed by any one acquainted with the whole of the properties of the things, but not specially interested in any.

The *object* of a classification into such groups is the best possible ordering of our ideas in reference to the things, or to make us think of those objects together which have the greatest number of important common properties.

Its *general problem* is to provide that the things be thought of in such groups, and those groups in such an order, as will best conduce to the ascertainment and remembrance of their laws.

II. General Principles which should govern the formation of a natural classification.

All Real Kinds are Natural Groups; and a Natural Classification must incorporate into itself all distinctions of Real Kind in the objects with which it is concerned.

The Infimæ Species—the lowest classes,—in a Natural Classification, will (almost invariably) be the logical infimæ species,—that is, the lowest Real Kinds.

The next step is to class these into higher groups. Certain species are, in the first instance, suggested to us by a feeling of general resemblance (*i.e.*, by type) as being allied; we then determine *what characters* these resembling species have in common, and by means of some of these we constitute our *genus;* and so on with the still higher groups. It is not, however, absolutely essential that *all* the characters assigned to the higher groups should be found in every lower group contained therein; it is sufficient if any lower group contains enough of them to cause it to resemble the members of that higher group more than of any other.

Upon what principle ought we thus to select characters for forming our groups?

We must select such characters as will constitute our groups, so that the members thereof shall possess the *greatest number* and the *most important* of their properties in common.

To this end one or both of the following requisites must be fulfilled; and in proportion as they are fulfilled is the excellence of the classification:—

(1.) The selected characters must themselves be important properties.

(2.) They must be marks of other properties, numerous and important.

Therefore, if we can, we should select as our distinctive characters the *causes* of many other properties, because — (1.) they are themselves important, and (2.) are the surest of marks. Again, *properties upon which the general aspect depends* should, *cæt. par.*, be selected; this, however, is not a *sine quâ non*.

III. How the names of Natural Groups should be constructed.

The names should convey, by their mode of construction, as much information as possible; they should have the greatest amount of independent significance which the case admits of.

There are two ways of giving a name this sort of significance:—

(1.) By making the name indicate, by its mode of formation, the very properties it is designed to convey; such as are sure marks (as chemical composition is) of all other properties. Chemical names are examples, as "protoxide of iron." This, however, is seldom practicable.

(2.) By making the name express the natural affinities of the group. This is accomplished by incorporating the proximate generic name with the specific, as "Felis leo." Even a ternary nomenclature, by incorporating the next higher generic name, has been used, as "Rhombohedral Lime Haloide."

IV. Whewell's theory that Natural Groups are constituted by *type, not by Definition*.

(The meaning of this is simply that objects are aggregated into Natural Groups on the basis of *mere general resemblance* (see p. 32), that is, what Whewell calls by reference to a type, and not by resemblance in specific assignable particulars which can be expressed in a definition.)

A "type" is an eminent example of any class, *i.e.*, an example which presents the characteristics of the class most conspicuously and completely. Natural classes, according to Whewell, are formed by being gathered round these types; and a class really consists of the type, and all objects which bear a certain amount of general resemblance to the type.

Mill's criticism:—

Natural groups *are* determined by characters (*i.e.*, by Definition, which enumerates those characters), not by Type or mere general Resemblance; but there is this amount of truth in Whewell's view:—

1. It is not, as already said, necessary for every member of a natural group to possess *all* the characters laid down as those of the group; and so far the definition may be said to fail in determining the group. In fact, natural classes might be defined in this way —those things which either possess such and such characters (those enumerated in the definition), or resemble those things which do possess them more nearly than they resemble anything else.

2. Our general conception of the group *is* a type, to which we usually in the first instance refer as a ready means of *suggesting* to what group any given

individual or species will most probably belong; but a *determination* of the question must always rest upon a reference to the characters laid down in the definition of the group. Natural grouping may, then, be said to be *suggested* by type (*i.e.*, by mere general resemblance), but *determined* by definition (*i.e.*, by possessing specific characters or properties).

CHAPTER VIII.

Classification by Series.

Inasmuch as Zoology presents the best example of Classification by Series, it may be taken as the special example, and the phenomenon "Animal life" as the phenomenon we are supposed to wish to study.

I. The subject generally.

The object of Classification into Natural Groups is, as already stated, to make us think of those objects together which have the greatest number of important common properties, and which, therefore, we have oftenest occasion, in the course of *our Inductions generally*, for taking into joint consideration.

But when our object is to facilitate the inquiry into *some particular phenomenon*, more is required. The classification must then bring the objects together in such a manner that the simultaneous contemplation of them will throw most light upon that particular subject. We must arrange the various groups *into a Series*, following one another according to the degrees or perfection in which they severally exhibit the phenomenon. The phenomenon itself, therefore, must form the guiding principle of such an arrangement.

It is evident that this serial classification, according to the degrees of the phenomenon of which the laws are to be investigated, puts the instances into the order required by the Method of Concomitant Variations, which, as already pointed out, is often the only available method in the case of phenomena which we have but limited means of artificially separating.

II. The requisites of a classification of this kind are :—

1. To bring into one grand class all kinds of things which exhibit the phenomenon, in whatever variety of form or degree.

2. To arrange these kinds into a Series, according to the degree in which they exhibit it; beginning with those which exhibit it in the greatest intensity and perfection, and terminating with those which exhibit least of it.

Thus the phenomenon being, as supposed, "Animal life," the first step, after forming a distinct conception of the phenomenon itself, is to erect into one great class—that of "animals"—all the known kinds of objects in which that phenomenon presents itself. We must, in the next place, arrange the various kinds in a series, those which exhibit

animal phenomena in the highest degree, as man, at the top, and gradually decreasing as we go down.

It may happen that the arrangement required for the special purpose coincides with that required for general purposes; this will naturally happen when the special phenomenon we are studying is so important as to determine the main of the properties generally.

III. The assumption of a Type Species is indispensable in inquiries of this kind.

By a Type Species is meant that one amongst the different kinds which exhibits the property we are studying in the highest and most characteristic degree.

This assumption of a type is necessary, because:—

1. We must study the phenomenon in its highest manifestations, in order to qualify ourselves for tracing it through its less obvious forms—for recognising the identity of the phenomenon under all its variations.
2. Every phenomenon is best studied, *cæt. par.*, where it exists in the greatest intensity; it is then that effects, which depend either upon it or upon the same cause with it, will exist in the greatest degree.
3. The phenomenon, in its higher degrees, may be attended by effects or collateral circumstances which, in the smaller degrees, do not occur at all.

IV. How the internal distribution of a series may most properly take place,—in what manner it should be divided into orders, families, and genera.

1. The main principle of division, of course, must be natural; the classes formed must be natural groups.
2. But the principle of natural grouping must be applied in subordination to the principle of a natural series, this series having its arrangement determined by the variations in the particular phenomenon; breaking it into primary divisions, if possible, at the exact points where variations in the intensity of that phenomenon begins to be attended with conspicuous change in the general properties of the objects.
3. In like manner each primary division should be so subdivided that any one portion shall stand higher than the next below in respect of the special property, or set of properties, we are studying.

V. Finally, though the kingdoms of organised nature afford, as yet, the only complete example of scientific classification, and the animal kingdom the only complete example of classification by series, yet the same principles are applicable in all cases where mankind are called upon to bring the various parts of any extensive subject into mental co-ordination. The proper arrangement, for instance, of a code of laws must depend upon similar conditions.

BOOK V.

FALLACIES.

I. *Fallacies in general.*

A Fallacy is an argument in which inconclusive or apparent evidence is made the basis of a belief; and a catalogue of the varieties of apparent evidence (*i.e.*, evidence which, while seeming to be real and conclusive, is not so) is an enumeration of fallacies.

II. *We do not include amongst Fallacies—*

1. *Mere blunders*—errors arising from a casual l pse, like a mistake in working an arithmetical sum while the general mode of procedure is correct.

2. *Moral sources of error, which are:—*

(*a.*) Indifference.

(*b.*) Bias—the most common being bias by our wishes, but very frequently also by our fears.

The moral causes of error in reasoning, though most

powerful, are yet indirect; they are but *remote* causes, and can only act through the intellectual causes; and to guard against these last is to guard against every other source of error.

III. *Classification of Fallacies* (see first page of table).

The five great classes into which Fallacies are divided are:—

1. Fallacies of Simple Inspection.
2. „ Observation.
3. „ Generalisation.
4. „ Ratiocination (Syllogistic).
5. „ Confusion.

The propositions which are *not* evidence of a particular conclusion are of course innumerable, and no classification can be based upon that merely negative property; but we may base it upon the positive property of *appearing to be evidence;* and we may arrange fallacies either (1.) according to what makes the evidence appear to be evidence, not being so (as the fact of its not being distinctly understood), or (2.) according to the particular kind of evidence it simulates (Inductive or Deductive). Mr Mill's classification is based on these principles jointly.

As it is seldom that insufficient evidence, when clearly understood and unambiguously expressed, would not be seen to be fallacious, more or less of the element of Confusion enters into most fallacies; but the class "Fallacies of Confusion" is reserved

for those in which Confusion is the chief, if not the sole, cause of the error.

Mr Mill's classification may be briefly sketched out thus:—

First, where the conclusion is assumed without there being any evidence to support it,—where it is believed as a "self-evident axiom,"—"Fallacies of Simple Inspection," "*à priori* Fallacies," or "Natural Prejudices;" and *Second*, where there is some evidence—"Fallacies of Inference." This last is subdivided according as (1.) the evidence is not distinctly understood (*i.e.*, not clearly seen to be what it really is), which gives us the "Fallacies of Confusion;" and (2.) as the evidence is distinctly understood. This last is again subdivided according as the evidence consists of (*a.*) particular facts (Inductive), or (*b.*) general propositions (Deductive); and each of these is again subdivided according as (1.) the evidence is false, or (2.) is true, but inconclusive.

It must not be supposed that any given fallacy can always be referred absolutely to one or other of the great classes. Except Fallacies of Confusion, hardly any fallacy can be assigned to its proper place till it has been expressed at full length; and the mode of doing this is often a matter of choice. All that we can generally say, then, in any particular case, is, that *if* the intermediate steps in the argument be filled up in such and such a way, the fallacy will fall into such a class.

THE FALLACIES

are divided by Mr Mill into *five great classes*, thus:—

I. "Fallacies of Simple Inspection," or "à Priori Fallacies."

(Where a proposition is improperly accepted as being "Self-Evident;" received as an *à priori* truth which requires *no evidence*, upon a "Simple Inspection" of it as it were, *i.e.*, upon the mere comprehension of its meaning.)

II. Fallacies of Inference.	Evidence clearly conceived, and understood.	Inductive.	Evidence false.	Fallacies of Observation.
			Evidence true, but inconclusive	Fallacies of Generalisation.
		Deductive.	Evidence false.	Since the false evidence must either be particular facts or general propositions, this class must come under one of the three preceding heads.
			Evidence true, but inconclusive	Fallacies of Ratiocination.
	Evidence not clearly conceived..........................			Fallacies of Confusion.

I.—Fallacies of Simple Inspection.

The following are examples of some principal forms:—

(*a.*) That the Inconceivable is false.

(*b.*) That everything which can be conceived in the mind must have a corresponding real existence in fact. (*Realism* an exaggerated form of this Fallacy.)

(*c.*) The doctrine of the "Sufficient Reason," that a thing must be so and so, because we know of no reason why it should be otherwise.

(*d.*) That the distinctions in nature must correspond to distinctions in language. (Common error with Greek philosophers.)

(*e.*) That a phenomenon can have but one cause. (An error which vitiated Bacon's Principles of Inductive Inquiry.)

(*f.*) That there must be a resemblance between a phenomenon and its conditions.

II.—Fallacies of Observation.

Here the error lies in *overlooking* or in *mistaking* something (*i.e.*, in collecting our data), and therefore we have either:—

Fallacies of Observation.
- (*a.*) *Fallacies of Non-Observation.* (overlooking)
  - Of Instances.
  - Of Essential Circumstances.
- (*b.*) *Fallacies of Mal-Observation*—mistaking (seeing wrong) inference for perception—believing that we have an immediate knowledge of something which we really infer.

Non-Observation or neglect of instances may occur either (*a.*) *From the circumstance that some of the instances are naturally more impressive than others,*—as, for instance, positive against negative instances. We are very apt to notice instances in which a phenomenon occurs, without regarding the equally important instances in which it does *not* occur. (*b.*) *From preconceived opinion,*—the most fertile source of error of this kind. That which in all ages has made men unobservant of the plainest facts, is their being contradictory to first appearances or any received belief. Thus, for centuries it was universally held that a body, ten times as heavy as another, fell to the ground ten times as fast; that the magnet exerted an irresistible force, and so on.

III.—Fallacies of Generalisation.

Here we have rightly obtained the obtainable evidence bearing on the conclusion; but we have wrongly concluded from it. The error lies in making the Inference, not in collecting the data.

This class of Fallacies—the error of drawing conclusions from insufficient evidence—is the most extensive of all, as might indeed be anticipated. It is only possible, therefore, to indicate some of the principal sub-classes:—

Fallacies of Generalisation.

- (*a.*) Generalisations which *cannot* in the nature of the case be established, where we have no real data or evidence to argue from,—as, for example, inferences as to what may go on in remote parts of the universe.
- (*b.*) All propositions which assert impossibility (universal negative propositions), except those which assert mathematical truths or the impossibility of exceptions to the universal law of causation.
- (*c.*) All generalisations which profess to resolve radically different phenomena into the same.
- (*d.*) The fallacy involved in placing mere empirical laws (and those often of the lowest kind) on the same footing of generality as true casual laws. As—
  - (1.) Empirical laws generalised from mere negations. ("What *has* not happened, never *will*.")
  - (2.) Empirical laws arrived at merely by the "induction by simple enumeration."
- (*e.*) Generalisations which improperly infer causation. (*Fallacia non causa pro causâ.*)
- (*f.*) Arguments from false analogy. (*Fallacia non talis pro tali.*)

Avoid confusion between "Ind. by *Simple* enumeration;" and "Ind. by *Complete* enumeration;" in the former we conclude that a law is true simply because we have never met with an instance to the contrary; the latter is the same as the "Mere Verbal Transformations" of Mill.

IV.—Fallacies of Ratiocination.

Fallacies of Ratiocination.

1. Fallacies of *Immediate Inference* (as in the Conversion, Opposition, Æquipollency of Propositions).
2. Syllogistic Fallacies (= the "*Logical Fallacies*" of Whately). Undistributed middle, illicit process of major and minor, and so on.
3. "Changing the Premisses" — *Secundum quid.* / *Per accidens.*

The meaning of the phrase, "Changing the Premisses," applied by Mill to a certain class of the Fallacies of Ratiocination, is this:—A premiss in a syllogism is regarded as being the conclusion of some previous act of inference. Now, if the proposition, *as laid down for a premiss*, is really distinct from that which was proved, an error may easily arise in making a deduction from it,—*a change* is made in passing from the proposition *as a conclusion* to the proposition *as a premiss*. The *Fallacia a dicto secundum quid ad dictum simpliciter* (briefly designated "*secundum quid*"), and the *Fallacia accidentis* are important forms of this sub-class of Fallacies; in both cases the error lies in laying down a major premiss too absolutely or generally,—more generally, in fact, than the evidence which supports it will warrant. Thus, if we say, "*All men have a right to their personal liberty*," it is clear that, generally speaking, we should really mean to limit it in some such way as this,—"All men, who are of sound mind, and who are not guilty of criminal conduct, have a right to their personal liberty." The evidence for the proposition only proves this more limited form; and if we use it as a premiss without these tacit limitations, we may be guilty of a fallacy.

V.—Fallacies of Confusion.

Here the mistake lies, not so much in over-estimating the probative force of known evidence, as in the absence of a distinct and definite conception of what that evidence really is, or what conclusion is required to be proved.

Fallacies of Confusion are :—	1. Ambiguous language (the "semi-logical" of Whately).	F. *equivocationis.* F. *amphibolia.* F. *figuræ dictionis.* F. *compositionis.* F. *divisionis.* F. *plurium interrogationum.*
	2. *Petitio Principii.*	The employment of a proposition to prove that upon which it is itself really dependent for proof.
	3. Arguing in a circle.	Proving two propositions reciprocally from one another, or more than two in a reciprocal manner.
	4. *Ignoratio Elenchi.*	Proving part of a conclusion. Proving a conclusion vague from the use of complex and general terms. Fallacy of shifting ground. Fallacy of objections. Fallacy of special appeals as "*ad hominem*," &c.

The Fallacy, "*Ignoratio elenchi*,"—ignoring the elenchus —is the proving of a proposition resembling more or less the conclusion required, but not really identical with it,—a *very common* form of Fallacy. The elenchus being the contradictory of the assertion of the supposed opponent.

I.—Fallacies of Simple Inspection.

Here a proposition is either admitted as true upon a "simple inspection" of it, as a self-evident truth, without any extraneous evidence, or, perhaps, more commonly the case is that *à priori* considerations *only create a presumption* in favour of a proposition, so that it is accepted, not absolutely without evidence, but upon evidence which would be seen to be insufficient if the presumption did not exist.

Amongst the many forms in which such errors may be presented, are :—

1. *That the reality of a thing will follow the idea of it; that the idea is either a prognostic, or even a cause of the thing thought of.*

This is extensively exemplified in many popular superstitions: the Romans, for instance, would not mention unlucky words, as "death."

2. *That a wonderful or precious thing must have wonderful properties.*

Gold regarded as the universal medicine.

3. *Things we cannot help thinking of together must coexist.*

Thus, it is often argued that *B* must accompany *A* in fact, because *B* is involved in the idea of *A*. This argument is at most an appeal to the authority of our predecessors. The doctrine that whatever the idea contains must have its equivalent in the thing, pervades the philosophy of Descartes, Leibnitz, and Spinoza, and the modern German metaphysicians.

4. *The inconceivable is false.*

This has been already examined.

5. *That everything which can be conceived in the mind must have a real existence in fact.*

Realism was an exaggerated form of this fallacy,—arguing, because a general idea of "man" can exist in the mind, there must be something really existing corresponding to that idea, just as when we think of any particular man there is a real corresponding existence.

6. *The principle of the sufficient Reason.*

That is, a phenomenon must follow a certain law, because we can see no reason for its deviating from [illegible] Thus

begin to move, because if it did, it must move in some particular direction, and we can see no reason why it should move in one direction rather than another ∴ it will not move at all. But the fact of our being able to see no reason, is not always a proof that no such reason exists.

7. *That the differences in Nature correspond to our received distinctions in names and classifications.*

This fallacy prevailed to an extraordinary extent amongst the Greek philosophers, who imagined that by an analysis of words they could discover facts.

8. *That a phenomenon can only have one cause.*

This was the error which misled Bacon.

9. *That the cause or conditions of a phenomenon must resemble that phenomenon.*

This does sometimes happen,—motion may produce motion,—but very commonly no resemblance whatever can be traced between an effect and its cause.

II.—Fallacies of Observation.

The term "observation" is here equivalent to the ascertainment of the facts upon which an

whether by direct experience or by inference from something else.

A fallacy of observation, then, may be either *negative* or *positive;* negative or non-observation when all the error consists in overlooking something which might have been known, and which, if known, would make a difference in our conclusion; positive, mal-observation, when something is *not simply unseen, but seen wrongly;* when a fact or phenomenon, instead of being taken for what it really is, is mistaken for something else. And, as we have previously observed, the *senses* cannot properly be said to be capable of error, but only *inferences* from sensations, this kind of fallacy can only happen when something which has really been erroneously inferred is supposed to have been actually observed.

As regards *non-observation,* we may overlook either (1.) Instances, or (2.) Essential circumstances in those instances (see table). It may be added that neglect of instances does not *per se* and necessarily vitiate the conclusion, unless we at the same time neglect to eliminate chance, which error would come under the next head,—Fallacies of Generalisation.

III.—Fallacies of Generalisation.

In addition to what is given in the table, we may notice the following points:—

Generalisations which profess to resolve radically different phenomena into the same.

Whenever our consciousness recognises between two of its states *a radical distinction;* whenever we feel that no mere adding on of the one phenomenon to itself would produce the other (as it would if the difference were only in degree), the two states must be the result of the operation of radically different laws, and any attempt to resolve the one into the other must be futile. (See also Book III., chap. xiv.)

Undue extension of Empirical Laws.

As examples of the kinds of Empirical Laws which are often unduly extended, we have—

(1.) Empirical Laws generalised from mere negations—their formula being "whatever has never happened, never will;" as "negroes have never been so highly civilised as whites, therefore they never will," and such like.

(2.) Empirical Laws, though based on positive data, yet only established by an Induction *per enumerationem simplicem,* stand one degree higher in the scale, but still ought only to be extended to adjacent cases.

Generalisations which improperly infer causation—

The most common is the fallacy known as "the *post hoc ergo propter hoc,*" arguing that *B* is caused by *A*, because *B* follows *A*.

Arguments based on false analogy—

For the conditions which determine the probative force of an analogical argument, see Book III., chap. xx.

The most fertile source of fallacies of generalisation is *bad classification*,—bringing together under a common name things which have no common properties, or at least no peculiar common properties.

IV.—Fallacies of Ratiocination (see table).

[By Immediate Inference is meant the direct deduction of one Proposition from another or others, without the intervention of a middle term. The following are some of the common forms of this kind of Inference (they would come under Mill's class of "Inferences improperly so called").

1. *Immediate Inferences by conversion*—

All men are mortal;
∴ Some mortal beings are men.

2. *By opposition*—

It is true that all men are mortal;
∴ It is false that some men are not mortal.

3. *By added determinants*—

A negro is a fellow-creature;
∴ To murder a negro is to murder a fellow-creature.

4. *By fusion of judgments*—

A negro is a fellow-creature;
Honesty deserves reward;
∴ A negro who is honest is a fellow-creature deserving of reward.

It is evident that a fallacy *may* lurk in processes of this kind.]

Syllogistic fallacies include all which offend against the laws of Syllogism.

V.—Fallacies of Confusion.

Fallacies of ambiguous language—

Here the premisses are *verbally* sufficient to prove the conclusion, but not really; they are the same as the "semi-logical" of Whately.

Petitio Principii—

The employment of a proposition to prove that upon which it is itself really dependent for proof, by no means implies that degree of inattention or imbecility which might seem at first sight involved in such an error. We must remember that even philosophers hold many opinions without exactly remembering how they came by them, or upon what evidence they were based; and in such a case they may easily be betrayed into deducing from them the very propositions which are alone capable of serving as premisses for their proof.

Arguing in a circle—

Is an attempt to prove two propositions reciprocally from one another, or three or more propositions in a similar manner. Thus, *A* is true because *B* is, *B* is because *C* is, *C* is because *A* is. This form of error is, however, more frequent in the form of *simply admitting* two propositions which can only be proved from one another, than as a deliberate attempt to do this.

Of course, a proposition would not be admitted merely as a corollary from itself, unless it were so expressed as to *seem* different; this is often done by stating one proposition in the concrete, the other in the abstract form, or one in Saxon, and the other in Classical phraseology.

Ignoratio elenchi—

Ignoring the elenchus ("the elenchus" being the contradictory of the assertion of the supposed opponent), is the proving of a conclusion more or less like the one required, but not really identical with it. This is a very common form of fallacy.

BOOK VI.

LOGIC OF MORAL SCIENCES.

CHAPTERS I. II. AND III.

By the Moral Sciences we mean those relating to the human mind and to human society; these form the most complex problems which can be submitted for our consideration, and it remains in this book to determine the method of scientific inquiry most likely to lead to satisfactory results in connexion with these questions. But, first, it is necessary to obviate an objection that may be made to the effect that human actions are not the subject of law, and, therefore, not of Science.

Liberty and Necessity.

of the sequence between motives and actions, is simply this—given the motives present to the mind of an individual, and given also his character and disposition, the manner in which he will act may be unerringly inferred. This is proved by the universal experience of mankind; whenever we rely upon a human being acting in a particular manner, we rely upon the uniformity of the sequence between motives and actions; and a most convincing proof is presented by statistics, which show the uniformity of the occurrence of apparently casual acts, when we observe on a scale sufficiently large to eliminate chance. It is sometimes said that our consciousness proves to us that the will is free, meaning by this that its acts are spontaneous, uncaused. But consciousness testifies nothing of the kind, it only testifies that we are under no *compulsion;* but the law does not assert this,—it simply asserts that the act follows the motive causes by a certain and unconditional sequence, it is no more a question of *constraint* than in the case of physical causes and effects.

There may be, therefore, a Science of human nature. Such a Science cannot, however, be a Science of exact predictions, but only of tendencies, since the causes are too uncertain to enable us to go beyond this.

CHAPTERS IV. AND V.

Laws of Mind.

The laws of mind are the laws by which one state of mind is produced by another. The simple laws of

mind must be ascertained by experiment; the complex laws are results of these, either by way of composition of causes, or as Heteropathic Effects. The mental differences between individuals are generally not ultimate facts, but are the results of differences in the mental history, education, circumstances, &c.

Although mankind have not one universal character, yet there are universal laws of the formation of the character (laws of Ethology). These laws cannot be discovered experimentally; the Deductive Method is the grand agent, observation being only valuable as affording the means of verifying its conclusion; the object of the Science of Ethology being to determine from the general laws of mind, combined with external circumstances, the conditions which aid or check the growth of good or bad qualities. Education will then consist in applying these results.

CHAPTERS VI. VII. VIII. AND IX.

The Social Science.

Social phenomena, being the phenomena of human nature in masses, must obey fixed laws, since human nature is subject to the same. The Social Science can never be a science of positive predictions, but only of tendencies, like most of the propositions relating to human nature. There have been many attempts to investigate this science, and to build up systems. A consideration of two methods

erroneously used, the Experimental and the Abstract Deductive, may with advantage form a preface to the true method.

The Experimental or Chemical Method in the Social Science.

The followers of this method refuse to accept conclusions except they be based upon *specific experience* in all cases. This attempt must fail, for we have already pointed out that in complex effects, direct Induction is scarcely ever applicable. Here, on account of the number and complexity of the causes, this is pre-eminently true.

The Abstract Deductive or Geometrical Method.

Those philosophers who have applied this method in the treatment of questions of Social Science have been correct in so far as they have been aware that the method of that science must be Deductive, but have erred in taking the application of that method to sciences not concerned with causation (as Geometry) as the type of the method required here. Their usual plan has been to take some proposition or propositions as premisses or axioms, and from them to deduce and build up a system. This method was adopted by Hobbes and by Bentham; but it is not the true method. The Social Science is a science of causes, and causes may be counteracted, and hence its method must be that form of the Deductive Method which is applicable to such sciences, namely,—

The Concrete or Physical Deductive Method.

That is to say, we must compound with one another the laws of all the causes on which any effect depends, and infer its law from them all. It is true we must often invert the order of our proceedings, and first obtain our conclusions conjecturally from specific experience, and then verify them by *à priori* reasonings.

Sociology, we have already remarked, is a science, not of positive predictions, but only of tendencies; and not only so, but its assertions must be hypothetical, and state the operation of a given cause in *given circumstances.* It also answers best to divide the science into subordinate sciences,—each of which considers one great social cause. Thus, Political Economy considers society as influenced by the desire of wealth.

CHAPTERS X. AND XI.

Inverse Deductive or Historical Method.

By a "State of Society" is meant the simultaneous state of all the greater social facts or phenomena. Such are the degree of knowledge, of intellectual and moral culture, wealth, industry, social classes, laws, &c.

Now, amongst these various phenomena there are certain *uniformities of coexistence;* that is to say, it is

not any combination of these social facts which can coexist, but only certain combinations. Just a ... various parts and states of the individual body ... a constant reciprocal influence ove one another, so it is in the body politic; there is a *consensus* between the different social facts; and the study of these uniformities of coexistence constitutes the science of *Social Statics*.

But besides presenting phenomena of this kind, society is in a constant state of progress; the state of society at any given time differs from its state at some previous time; the study of the laws by which any state of society produces the state of society which succeeds and takes its place, constitutes *Social Dynamics*—the theory of society as progressive.

The evidence of history goes to prove that one great element is predominant over all others as the prime agent in determining social progress,—that is, *the state of the speculative faculties*, including the nature of the beliefs which men hold at the time, and the means by which they have arrived at them. And M. Comte has laid down one generalisation which he regards as the fundamental law of the progress of human knowledge,—viz., that speculation on every subject has three successive stages,—first, when the tendency is to explain phenomena by supernatural agencies ...

CHAPTER XII.

Logic of Practice or Art.

In speaking of the logical method of "*Art*," that term is used in the sense of a body of rules directed to some practical end, as when we speak of the "Art of Building," "of Government," and so on; and not as having reference to the poetical or æsthetic aspect of things.

Art, then, is characterised by expressing its propositions in the imperative mood; it speaks in rules or precepts, as contrasted with the direct indicative assertions of science.

The logical method of art may be summed up thus:—Every art starts from a single major-premiss—that such and such an end is desirable. Science, then, investigates the means by which the end can be secured; and this being accomplished, it hands over the necessary propositions to Art to be turned into practical rules.

In order to know what things are really desirable, we require a Science of Teleology, or of ends (*i.e.*, ...

APPENDIX.

I.

Connotative Names.

By the "Connotation" of a name is understood the attributes which we mean to assert that an object possesses, when we predicate the name of that object. Thus, if we assert that an object before us is a "man," we mean to convey that that object possesses certain attributes—animality, rationality, and two-handed, upright form. These attributes constitute the connotation or meaning of the name "man."

The mode in which some Logicians have represented the point is by speaking of the idea as "*comprehending*" or including other ideas; the idea of "man" would be said to *comprehend* the idea of animality, rationality, &c. What the idea comprehends is, in fact, precisely equivalent to what the name connotes; and the definition is spoken of as being the unfolding or stating in words either of the

Non-connotative Names

Are simply marks and nothing more—"non-significant marks." If every house in a town has its own letter or number of some kind on the door, such a number or letter would be a mark of the corresponding house, but it would signify nothing, convey no meaning. Such exactly are non-connotative names,—the chief of which are proper names, and the names of simple attributes. "Cæsar" is like a cross put on an individual, chiefly to identify him, and save the trouble of a long description; but it conveys no meaning. If we are told that an object is called "Cæsar," we should know from that nothing of its properties or attributes; it might be a man, dog, horse, &c. But if instead of speaking of an object as "Cæsar," we speak of it as a "Roman general," this is not only a name, but a name significant of something,—viz., belonging to Rome and being a general, and therefore is connotative.

The following classes include the most important Connotative Names:—

1. *All concrete general names*,—as "man," "animal," "planet,"—the names of classes of objects. Such a name evidently connotes the attributes, the possession of which makes any object a member of the corresponding class.

2. *Descriptive individual names*—that is, names which, instead of designating an individual by a mere unmeaning proper name, point him out by some qualities or properties or marks which belong to him. Thus the name "Gladstone" is a *mere* mark, and therefore a non-connotative name, while "present Prime Minister of England" refers to the same individual, but is connotative, *i.e.*, it implies attributes or properties. *The former name conveys in itself no information;* anything which we happen to know of the person when the name is pronounced is merely accidental; but the latter does tell something, wholly independent of such casual knowledge, and equally to every person, however well or ill informed with respect to the individual in question. Some authors on Logic (see "Shedden's Manual," p. 17, &c.) have maintained that proper names are connotative; by this they mean that, for example, "Gladstone" connotes "a politician, in 1869 Prime Minister of England," &c., because such circumstances may be brought to mind by the mention of that name; so that they hold that proper names connote whatever any given individual in whose hearing they are pronounced may happen to know of the person to whom the name belongs. The name "John," for instance, being universally and exclusively applied to males, would, according to this view, connote to every one the attribute "masculinity,"—the being of the male sex; while to any individual who happened to know the particular John referred to, it would connote anything whatever that he might happen to know or remember about him. Now, to take the stronger of these cases,—that is, where the circumstance associated with the name is of such a character that the mention of the name would suggest the circumstance to every one, as the name "John" would that the person spoken of was a male. The name John, then, we are to suppose, connotes "being of the male sex;" but every name whose (entire) connotation

consists of a given attribute or set of attributes, is the name of a class of objects,—viz., those objects which possess the connoted attributes,—and to every one of these objects the name is applicable.* The name John, however, is not the name of every individual of the male sex, and this consideration shows conclusively that the attribute "masculinity" does not constitute the connotation of that name.

The distinction, in *fact*, is obvious enough between what a name *really means*, and what we may happen to know in some way or other about the object to which it is applied. Of course the term "connotation" may have its meaning extended to include such casual associations, but if we assign *such* a meaning to it, we must remember that it is then *totally* distinct from the connotation which Mr Mill so constantly refers to. Accidental knowledge of the sort we are discussing is of no importance in Logic, and to incorporate it with true connotation would destroy all the value of the distinction, and constitute a mischievous distortion of the recognised signification of the word.

2. *Certain Abstract Names*,—or names of attributes, viz., those abstract names which are names either—(1.) Of attributes which have attributes; or (2.) Of groups or aggregates of attributes. Thus, "civilisation" is the name of an attribute (the corresponding concrete being "civilised beings"), which includes a number of other attributes,—a group, in fact, bound together by the name,—such as intellectuality, moral and æsthetic cultivation, and so forth, and these attributes form the connotation or meaning of the name "civilisation." As an example of an attribute which possesses attributes, Mr Mill gives "faultiness,"—the name of some quality which has the attribute "causing inconvenience," which, therefore, the abstract name "faultiness" connotes.

Non-connotative Names require but little illustration,—they are simple marks without meaning. The chief classes are—(1.) Proper names; and (2.) The names of simple, unanalysable attributes; or, in other words, of our elementary feelings. "Whiteness," for example, is a mark put on a certain quality of objects, just as "Cæsar" is a mark put on a certain individual.

By the *Denotation* of any name is understood the whole collection or aggregate of objects to which the name is applicable. The denotation of "man" includes every human being; of "law," every law; of "crime," every criminal act; and so in every case. Many logicians use the term "extension" of a name as equivalent to its denotation.

* We say "entire" for this reason,—suppose a name to connote attributes *A*, *B*, and *C*, and nothing more, then to *every* object which possesses the three attributes *A*, *B*, and *C*, the name is applicable, but not necessarily to objects which possess only one or two of these, as is indeed self-evident.

II.

Process of forming general notions (concepts, general ideas).

Take the name "man,"—in connexion with this we have three things,—the *name* itself, the *class* to which it corresponds (*i.e.*, its denotation), and the general *notion* or idea which is raised up within our minds by the mention of the name.

What is the nature of the process by which any such general notion is obtained, or by which such a class is formed? Let us place ourselves in the position of the first intelligent observer of nature; he would be continually encountering a variety of objects, and after a little experience he could not help noticing that certain of these resemble each other, the resemblance consisting in the possession of certain common attributes. Thus he would meet with objects which we now recognise as forming the class "*liquids*;" he would, on instinctively comparing such together, find that they agreed in possessing *certain* properties (perfect molecular mobility and [illegible]), and whenever he met with a

sisting of all objects which possessed them, and a general name might be imposed on that class, which name would connote or imply those same properties,—that is, the applicability of the name to any given object would depend upon that object possessing them.

It is evident, moreover, that the name, once given, serves ever afterwards to bind and keep together that group of attributes; were it not for the name we should be almost sure, sooner or later, to forget our classification, and have to make it over again, and even if not, we could not permanently register its results to communicate to others, or transmit to our successors.

The process, then, of forming general notions may be summarised thus:—

1. The *senses, and memory* reproducing their impressions, are continually giving us a knowledge of a succession of different objects.
2. *Comparison* of certain of these objects shows that they are similar, and we recognise the similarity as consisting in the possession of certain common attributes.
3. Our attention being thus concentrated upon these common qualities, the mind instinctively binds them up into an aggregate or group, which forms our idea of the class, that is, a general notion.

Different views as to the form in which general notions exist in the mind.

A general notion may be defined, as we have seen, as a conception of a multitude of individuals as an assemblage or class. Such general notions certainly exist in the mind in some form; we *can*, somehow, conceive of a multitude of individual objects as an assemblage or class, or we could not use general names with any consciousness of meaning.

Before giving a summary of the different views which have been held on this question, we may remark, that it is not strictly any part of Logic; it is sufficient for its purposes that the name of a class call up *some* idea by which we can, to all intents and purposes, think of the class as such. Mr Mill, however, gives the following different doctrines on the question. As to the nature of the idea called up by a general name:—

(*a*.) Doctrine of Locke, Brown, and the conceptualists,—that a general idea is composed of the various circumstances in which all the individuals denoted by the general name agree, and of no others.

(*b*.) Doctrine of James Mill,—that such an idea is that of a miscellaneous assemblage of the individuals belonging to the class. Thus, the name "man" is supposed to call up the idea of an assemblage or mass of human beings.

(*c*.) Doctrine of Berkely, Dugald Stewart, and the modern Nominalists.—that the idea of a class is really the idea of some one individual of that class with his individual peculiarities, but with the accompanying knowledge that such peculiarities are

not found in every member of the class,—that is, are not properties of it.

(*d*.) Bailey's view is, that the general name raises up an image, sometimes of one known individual of the class, sometimes of another; not unfrequently of several such individuals in succession, and sometimes an image made up of elements from different objects.

(*e*.) In a very large number of cases, where a general name is mentioned, *no* distinct idea whatever is called up in the mind; the name is used as a *mere symbol*, employed as an x or an a in an algebraical process.

It is impossible to discuss the subject fully here; it is only necessary to say, that the ideas called up by general names are certainly not always of the same kind. (Comp. "Symbolical and Notative Conception"—Thomson's Outlines.)

THE END.

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